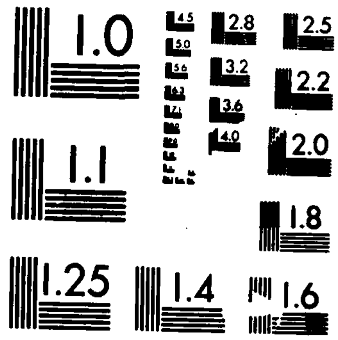
AD-A183 060 CODED FH/SS (FREQUENCY-HOPPED SPREAD-SPECTRUM) 1/1
COMMUNICATIONS IN THE PRES..(U) NAVAL RESEARCH LAB
WASHINGTON DC E GERANIOTIS ET AL. 30 JUN 87 NRL-9039

UNCLASSIFIED F/G 17/4 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Naval Research Laboratory
Washington, DC 20375-5000

2

NRL Report 9039

Coded FH/SS Communications in the Presence of Combined Partial-Band Noise Jamming, Rician Nonselective Fading, and Multiuser Interference

EVAGGELOS GERANIOTIS

Locus, Inc.
Alexandria, Virginia

and

University of Maryland
College Park, Maryland

and

JEFFREY W. GLUCK

University of Maryland
College Park, Maryland

Communication System Engineering Branch
Information Technology Division

June 30, 1987

87 7 ... 0 2

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS A183 068
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Report 9039	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b. OFFICE SYMBOL (If applicable) Code 5520	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Research 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER Contract No. N00014-86-C-2016 Grant No. N00014-85-G-0207
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61153N PROJECT NO. RR021-0542 TASK NO WORK UNIT ACCESSION NO. DN480-557

11. TITLE (Include Security Classification)
Coded FH/SS Communications in the Presence of Combined Partial-Band Noise Jamming, Rician Nonselective Fading, and Multiuser Interference

12. PERSONAL AUTHOR(S) Geraniotis, Evaggelos and Gluck, Jeffrey W.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Interim	FROM 5/85 TO 8/86	1987 June 30	35

16. SUPPLEMENTARY NOTATION
This research was performed on site at the Naval Research Laboratory.
J.W. Gluck is an NRL Fellow under ONR Grant No. N00014-85-G-0207.

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	(See page ii)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In this report, we address the problem of combatting combined interference in spread-spectrum communication links. We consider frequency-hopped spread-spectrum systems with M-ary FSK modulation and noncoherent demodulation that use forward-error-control coding. The interference consists of partial-band noise jamming, nonselective Rician fading, other-user interference, and thermal noise. The coding schemes that we analyze include: Reed-Solomon codes (with or without diversity and error/only, erasure-error or parallel erasure/error decoding); binary, nonbinary, and dual-k convolutional codes with or without side information (information about the state of the channel); and concatenated schemes (Reed-Solomon outer codes with either inner detection-only block codes or inner convolutional codes). In all cases, we derive the minimum signal-to-jammer energy ratio required to guarantee a desirable bit error rate as a function of the fraction of the band that is jammed when the number of interfering users is fixed, and the maximum number of users that can be supported by the system as a function of the fraction of the band that is jammed, when the signal-to-jammer energy ratio is fixed.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Jeffrey E. Wieselthier	22b. TELEPHONE (Include Area Code) (202) 767-3043 22c. OFFICE SYMBOL Code 5521

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted
All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

*U.S. Government Printing Office: 1985-507-047

18 SUBJECT TERMS (*Continue on reverse if necessary and identify by block number*)

Jamming	Frequency shift keying	Error correction codes
Antijamming	Multiuser interference	Convolutional codes
Spread spectrum	Fading	Reed-Solomon codes
Frequency hopping		

CONTENTS

CODED FH/SS COMMUNICATIONS IN THE PRESENCE OF COMBINED PARTIAL-BAND NOISE JAMMING, RICIAN NONSELECTIVE FADING, AND MULTIUSER INTERFERENCE

1. INTRODUCTION

Recently, there has been an increasing interest in the design and performance analysis of frequency-hopped spread-spectrum (FH/SS) systems because they can combat hostile interference and provide multiaccess capability. References 1 through 7 describe FH/SS systems that operate in environments characterized by several distinct types of interference including partial-band noise jamming, nonselective fading, other-user interference, and background (thermal) noise.

A common characteristic of the work described in Refs. 1 through 7 is that, although the effects of combined jamming and fading or combined other-user interference and fading on FH/SS systems have been studied, the effects of combined hostile and other-user interference have not been investigated.

This report describes the performance of FH/SS systems with binary or M-ary FSK modulation and noncoherent demodulation. These systems use forward-error-control (FEC) coding and operate in a combined partial-band noise jamming, other-user interference, Rician nonselective fading, and additive white Gaussian noise (AWGN) environment. Such a situation may arise in packet radio networks that use FH/SS signaling. We address problems related to the performance of a link in such a network. In particular, if the maximum number of transmitters operating in the vicinity (hearing range) of a receiver is a parameter of the network (which has been determined by some other network specifications), it is important to know the signal-to-jammer energy ratio that is required to guarantee a desirable bit error rate at the receiver in question. Similarly, when the signal-to-jammer energy ratio is a system parameter, we are interested in knowing what is the maximum number of transmitters in the hearing range of a receiver that yields a bit error rate below a prespecified, tolerable level.

The report is organized as follows. The system and channel model are described in detail in Section 2. Then, in Section 3 the performance of several FEC coding schemes is analyzed. In particular, Reed-Solomon (RS) codes with error-only, erasure/error, and parallel decoding are studied in Section 3.1. Binary and M-ary repetition codes with and without side information (i.e., information about the state of the channel: the presence or absence of hostile or other-user interference) are considered in Section 3.2. Binary and nonbinary convolutional codes and dual-k convolutional codes with and without side information are examined in Sections 3.3 and 3.4. Concatenated codes with inner detection-only block codes or binary or dual-k convolutional inner codes and RS outer codes with error-only, erasure/error, and parallel decoding are analyzed in Section 3.5. Finally, RS codes with time diversity and errors-only, erasures/errors, and parallel decoding are investigated in Section 3.6. In all cases, only hard decisions on the channel output are implemented. In Section 4, numerical results are presented on the minimum required signal-to-jammer energy ratio and the maximum number of transmitting users for all the FEC coding schemes enumerated above and comparisons are made. Finally, in Section 5 the key results of the report are summarized and conclusions are drawn.

Manuscript approved December 22, 1986.

2. SYSTEM AND CHANNEL MODEL

The FH/SSMA system model considered is that of Ref. 1 in an environment characterized by partial-band jamming and Rician nonselective fading. M-ary FSK data modulation with noncoherent demodulation is used. The hopping rate is no larger than the data rate (slow-hopping). N_s M-ary symbols (and thus $N_b = N_s \log_2 M$ bits) are transmitted during each hop (dwell time), where $N_s \geq 1$.

It is assumed that in the vicinity of a particular receiver there are K asynchronous transmitted signals, all of which share the same channel. It is also assumed that the receiver can acquire synchronization with the frequency-hopping pattern and the time of one of the K signals. The other K-1 signals can then interfere with the reception of the signal that was singled out. Our model of other-user interference is that of Ref. 1. There is no restriction on the power levels (or the communication range) of the transmitted signals. This is because we use bounds on the conditional probability of a receiver error when other-user interference is present that are independent of the power levels, phase angles, or time delays of the interfering signals.

The model for the partial-band noise interference is the one commonly used in the literature (e.g., Refs. 2 through 6), except that thermal noise [modeled as additive white Gaussian noise (AWGN)] is also assumed to be present at the receiver. Therefore, if N_J denotes the effective one-sided spectral density of the partial-band Gaussian noise (i.e., $N_J = P_J/W$, where P_J, the power available to the jammer, is assumed to be fixed and W is the total bandwidth of the FH/SS system), N_0 denotes that of the AWGN, and ρ $(0 \leq \rho \leq 1)$ is the probability that a particular dwell time (frequency slot) is jammed, then the one-sided spectral density of the Gaussian noise is

$$N_0 + \frac{N_J}{\rho}$$

with probability ρ, and it is N_0 with probability $1-\rho$. The density of the noise remains constant over the duration of the frequency slot (dwell time). Different dwell times are jammed independently.

The nonselective Rician fading channel model is that of Refs. 3 and 8. The received signal consists of a nonfaded component and an attenuated, phase-shifted, faded component (termed scatter component) whose delay with respect to the nonfaded component is negligible. The amplitude of the received signal has a Rician distribution, and the probability of error of an M-ary FSK system with noncoherent demodulation is given by (see Ref. 3):

$$P_{e,M}(\eta) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1+m\beta(\eta)} \exp\left[-\frac{m\delta(\eta)}{m+1+m\beta(\eta)}\right], \tag{1}$$

where $\beta(\eta) = \Lambda(\eta)/(1+\gamma^{-2})$, $\delta(\eta) = \Lambda(\eta)/(1+\gamma^2)$, and $\Lambda(\eta) = \bar{E}_b \log_2 M/\eta$ is the received signal-to-noise ratio, i.e., $\bar{E}_b$ is the received energy per information bit, and η is the spectral density of the AWGN. (For coded systems, $\Lambda(\eta) = r\,\bar{E}_b \log_2 M/\eta$, where r is the code rate.) Finally, γ^2 is the ratio of the expected relative strength of the scatter component to the expected relative strength of the nonfaded component. Notice that $\gamma^2 = 0$ implies that $\beta(\eta) = 0$ and $\delta(\eta) = \Lambda(\eta)$, so the Rician fading channel reduces to an AWGN channel. Similarly, $\gamma^2 = \infty$ implies that $\beta(\eta) = \Lambda(\eta)$ and $\delta(\eta) = 0$, so the channel becomes a Rayleigh fading channel.

3. PERFORMANCE EVALUATION OF CODED SYSTEMS

In this report we consider several forward error-control coding schemes, in particular Reed-Solomon (RS) codes, binary and nonbinary convolutional codes (CC), dual-k convolutional codes, several concatenated coding schemes (e.g., block inner code/RS outer code, binary CC inner code/RS outer code, dual-k CC inner code/RS outer code), and Reed-Solomon coding schemes with time diversity are analyzed.

3.1 Reed-Solomon Codes

We consider three distinct cases: (1) error correction, (2) erasure/error correction, and (3) parallel erasure/error correction, as in Ref. 5. In case (1), there is no information about the state of the channel; thus, the RS decoder attempts only to correct the errors. In case (2), we assume that channel monitoring provides information about the state of the channel (presence or absence of jamming or other-user interference). This information can be used by the RS decoder to erase the symbols that have suffered heavy interference. In this case, the decoder attempts to correct the erasures and the few errors that result from thermal noise. In case (3), the decoder corrects erasures and errors caused by thermal noise when the number of observed erasures is less than or equal to $e = n - k$; otherwise, it only attempts to correct errors. (See Ref. 5 for a detailed description of this algorithm when partial-band jamming is the only form of interference in the channel.)

Next, we evaluate the average error probability for the cases described above. We assume that RS codes over GF(M^m) are used; thus, there are m M-ary symbols in each RS symbol. Thus there are N_s/m RS symbols transmitted in each hop. For case (1), we upper-bound the probability of a symbol error for the uncoded system by

$$p_s \le 1 - (1 - P_h)^{K-1}[(1 - \rho)(1 - P_0)^m + \rho(1 - P_{J,0})^m]. \tag{2}$$

In Eq. (2), P_h denotes the probability of a hit from another user (i.e., both users use the same frequency slot for part of their dwell times). The probability of a hit for any RS symbol and M-ary FH/SS asynchronous systems has been shown in Ref. 1 to be upper-bounded by

$$P_h = \left(1 + \frac{m}{N_s}\right)\frac{1}{q}. \tag{3}$$

(The m in the numerator accounts for the fact that each RS symbol contains m M-ary FSK symbols, thus it is more likely to be hit than a single M-ary FSK symbol), and q is the number of available frequency slots. The frequency-hopping patterns are assumed to be memoryless random sequences (see Ref. 1) and independent for distinct users. The probabilities P_0 and $P_{J,0}$ in Eq. (2) denote the error probabilities of an M-ary FSK system with noncoherent demodulation disturbed by AWGN of one-sided spectral densities N_0 and $N_0 + (N_J/\rho)$, respectively. Thus $P_0 = P_{e,M}(N_0)$ and $P_{J,0} = P_{e,M}\ [N_0 + (N_J/\rho)]$, where $P_{e,M}(\cdot)$ is defined in Eq. (1). In Eq. (2), $(1 - \rho)(1 - P_0)^m + \rho(1 - P_{J,0})^m$ is the probability of no error due to Gaussian noise in m M-ary symbols, and $(1 - P_h)^{K-1}$ is a lower bound on the probability of no error due to any of the other $K - 1$ users. Notice that the conditional probability of error (given that a hit from another user occurred) has been upper-bounded by 1; in this way, p_s does not depend on the power levels, time delays, or phase angles of the different users. For $M = 2$ (binary FSK modulation), it has been shown in Ref. 10 that this conditional error probability is upper-bounded by 1/2; however, no such result has, so far, been established for $M > 2$.

When bounded distance decoding of RS codes with hard decisions is used, the symbol error probability for the coded system is given by (See Ref. 11):

$$P_{e,s} = \sum_{j=t+1}^{n} \frac{j}{n}\binom{n}{j} p_s^j(1 - p_s)^{n-j}, \tag{4}$$

where $t = \lfloor (n - k)/2 \rfloor$ is the error-correction capability of the RS(n,k) code (k information symbols in a codeword of length n). Equation (4) is valid when all of the RS symbols in the same codeword are subject to independent errors. This can be achieved by interleaving to a depth of N_s/m, so that only one RS symbol of any codeword is transmitted on each hop. Equation (4) can serve as an upper bound for the M-ary symbol error probability and the bit error probability of the coded system.

For case (2) the probability of an erasure is

$$\epsilon_s = \rho + [1 - (1 - P_h)^{K-1}] - \rho[1 - (1 - P_h)^{K-1}], \tag{5}$$

since we assume that the decoder erases a symbol if the jammer is present and/or if interference from other users is present. Thus the probability of a symbol error is

$$p_s = [1 - (1 - P_0)^m](1 - \epsilon_s), \tag{6}$$

since $1 - \epsilon_s$ is the probability of no interference from the jammer or from the other users, and $1 - (1 - P_0)^m$ is the probability of error due to the thermal noise alone. Notice that neither ϵ_s nor p_s depends on the signal-to-jammer ratio E_b/N_J. In this case, the probability of a RS symbol error at the decoder, as shown in Ref. 9, is

$$P_{e,s} = \sum_{\substack{j+l \le n \\ e+1 \le 2l+j}} \frac{j+l}{n} \binom{n}{j} \binom{n-j}{l} p_s^l \epsilon_s^j (1 - p_s - \epsilon_s)^{n-l-j}, \tag{7}$$

where $e = n - k$ is the erasure-correction capability of the RS code.

For case (3), we can apply the parallel erasures/errors decoding algorithm of Ref. 5, where partial-band noise jamming is the only source of interference, and that of Ref. 7, where partial-time jamming and thermal noise are the sources of interference, to the case in which partial-band noise jamming, multiple-access interference, Rician nonselective fading, and thermal noise are present.

When the number of erasures is less than or equal to $e = n - k$, the erasure correction capability of the code, the contribution to the decoder's error probability, is

$$P_{e,s;1} = \sum_{j=0}^{e} \binom{n}{j} \epsilon_s^j (1 - \epsilon_s)^{n-j} \sum_{\substack{e+1 \le 2l+j \\ l+j \le n}} \frac{j+l}{n} \binom{n-j}{l} P_0^l (1 - P_0)^{n-j-l}, \tag{8}$$

where the probability of an erasure ϵ_s is defined in Eq. (5), and P_0 is the error probability of an M-ary symbol caused by thermal noise. In Eq. (8), j is the number of erased symbols, $n - j$ is the number of symbols that are not erased, and l is the number of symbols out of those $n-j$ symbols that result in a receiver error due to the thermal noise alone.

When the number of erasures is larger than $e = n - k$, the contribution to the decoder's error probability becomes

$$P_{e,s;2} = \sum_{j=e+1}^{n} \binom{n}{j} \epsilon_s^j (1 - \epsilon_s)^{n-j}$$

$$\sum_{\substack{t+1 \le l_1+l_2 \\ l_1 \le j \\ l_2 \le n-j}} \frac{l_1 + l_2}{n} \binom{j}{l_1} \bar{p}^{l_1} (1 - \bar{p})^{j-l_1} \binom{n-j}{l_2} P_0^{l_2} (1 - P_0)^{n-j-l_2}. \tag{9}$$

In Eq. (9), $\bar{p}$ denotes the probability of error given that there is partial-band or multiple-access interference. It is specified by

$$\bar{p} = \frac{\epsilon_1}{\epsilon_s}[1 - (1 - P_{J,0})^m] + \frac{\epsilon_2}{\epsilon_s}\left(1 - \frac{1}{M^m}\right), \tag{10}$$

where ϵ_1 denotes the probability of being jammed but not hit by other users, and ϵ_2 denotes the probability of being hit by other users. These quantities are given by

$$\epsilon_1 = \rho(1 - P_h)^{K-1} \tag{11a}$$

and

$$\epsilon_2 = 1 - (1 - P_h)^{K-1}. \tag{11b}$$

Furthermore, in Eq. (9) j is the number of symbols in an RS codeword that are subject to either partial-band interference or multiple-access interference, whereas $n - j$ is the number of symbols subject only to AWGN. Then l_1 out of the j symbols that are subject to interference are received in error, whereas $j - l_1$ are not, and l_2 out of the $n - j$ symbols subject to only AWGN are received in error, whereas $n - j - l_2$ are received correctly. Therefore, the decoder commits an error when the total number of errors $l_1 + l_2$ exceeds t, the error-correction capability of the RS code. Finally, the total error probability at the output of the RS decoder is given by $P_{e,s} = P_{e,s:1} + P_{e,s:2}$.

3.2 Repetition Codes

We consider two cases: (1) when channel monitoring reveals the presence or absence of interference (other than AWGN) in the channel, and (2) when no information about the state of the channel is available. Both binary and M-ary FSK modulation with noncoherent demodulation are examined. In the case of binary FSK, the N_b bits of a dwell time are interleaved. In the second case, the $N_s = N_b/\log_2 M$ M-ary symbols are interleaved.

When information about the state of the channel is not available, majority vote decoding (where the decoder decides in favor of the symbol that was received the largest number of times) with hard decisions is the maximum-likelihood decoding algorithm. For binary repetition codes of block length n, the bit error rate (BER) is

$$P_{e,b} = P_2(n;p) = \begin{cases} \displaystyle\sum_{l=(n+1)/2}^{n} \binom{n}{l} p^l(1-p)^{n-l} & ; n \text{ odd} \\ \displaystyle\sum_{l=n/2+1}^{n} \binom{n}{l} p^l(1-p)^{n-l} + \frac{1}{2}\binom{n}{n/2}[p(1-p)]^{n/2} & ; n \text{ even} \end{cases} \tag{12}$$

where p denotes the error probability for a binary channel with combined multiple-access interference, partial-band jamming, Rician nonselective fading, and thermal noise and is obtained from Eq. (2) by using $m = 1$ and $M = 2$. For M-ary repetition codes ($M > 2$), the symbol error probability is obtained from

$$P_{e,s} = P_M(n;p) = 1 - \sum_{i=0}^{n-1} a_i p^i (1-p)^{n-i}, \tag{13}$$

where the coefficients a_i for n smaller than 10 are provided in the Appendix A of Ref. 6, and p is as above.

When information about the state of the channel is available, the requirement for maximum-likelihood decoding results in a complicated rule. For $M = 2$, it is described in Ref. 6; for $M > 2$, the decoding rule becomes too complicated to be useful for implementation.

Therefore, we consider the following suboptimal rule. For moderately large values of E_b/N_0 (the signal-to-AWGN ratio), it performs very close to that of the optimal (maximum-likelihood) decoding rule (see Appendix B of Ref. 6). Assuming that information about the state of the channel is available (presence or absence of interference other than AWGN) for each code symbol, the decoder counts the number of symbols hit by interference (multiple-access or jamming). If this number is equal to n, it executes majority vote decoding of all n symbols; if the number is smaller than n, it executes majority vote decoding of the symbols that were not hit by interference. For the channel model considered in this report, the bit error rate for this decoding scheme is expressed as

$$P_{e,s} = \bar{P}_M(n) = \epsilon_s^n P_M(n;\bar{p}) + \sum_{l=0}^{n-1} \binom{n}{l} \epsilon_s^l (1-\epsilon_s)^{n-l} P_M(n-l;P_0). \tag{14}$$

In Eq. (14), ϵ_s denotes the probability that a symbol is hit by multiple-access or partial-band interference and is given by Eq. (5); $\bar{p}$ denotes the symbol error probability given that interference is present and is specified by Eqs. (10) and (11) where P_h is defined by Eq. (3) for $m = 1$ (N_b should replace N_s for $M = 2$). The quantity $P_M(\cdot;\cdot)$ can be obtained from Eq. (12) and Eq. (13) for $M = 2$ and $M > 2$, respectively. For $M = 2$, Eq. (14) provides the bit error probability; for $M > 2$, it provides the symbol error probability and can serve as an upper bound for the bit error probability.

3.3 Binary and Nonbinary Convolutional Codes

For binary convolutional codes (CCs), binary FSK is used ($M = 2$), and all binary FSK symbols within the hop (dwell time) are assumed to be interleaved (the interleaving depth is N_b). The input to the Viterbi hard decision decoder consists of bits with error probability p given by Eq. (2) for $m = 1$, $M = 2$, and $P_h = [1 + (1/N_b)]1/q$. The bit error probability of the coded system, given in Ref. 12, (for CCs with rates b/n) is

$$P_{e,b} \le \frac{1}{b} \sum_{j=d_{free}}^{\infty} w_j P_j, \tag{15}$$

where d_{free} is the free distance of the code, P_j is the probability of the error event that the decoder chooses a path at distance j from the correct path, and w_j is the total information weight of all sequences which produce paths of weight j. The weights for binary CCs of various rates can be found in Refs. 12 and 13, and for nonbinary CCs in Ref. 9. For binary (or nonbinary) codes, P_j defined above coincides with the error probability of a binary (or nonbinary) repetition code of length j. Therefore, if there is no side information (i.e., channel monitoring to reveal the presence or absence of interference), P_j is given by Refs. 8 and 12 as

$$P_j = P_M(j;p) \le (M-1)\, Q_M(j;p). \tag{16}$$

Here $P_M(\cdot;\cdot)$ is defined in Eq. (12) for $M = 2$ (binary codes) and in Eq. (13) for $M > 2$ (nonbinary codes), and $Q_M(j;p)$ denotes the probability of error between two codewords of an M-ary repetition code of rate $1/j$ when side information is absent. The inequality in Eq. (16) follows from an application of the union bound. It has the advantage that it involves $Q_M(\cdot;\cdot)$ which for large j (and since $j \ge d_{free}$ it can be much larger than 10) is easier to compute than $P_M(j;p)$ (which is provided in Ref. 6 only for $j \le 10$).

In the absence of side information, the probability of error between two codewords of an M-ary repetition code of rate $1/n$ is given by Ref. 6 as

$$q_n = Q_M(n;p) = \sum_{\substack{j<k \\ j+k \le n}} \binom{n}{j} \binom{n-j}{k} (1-p)^j \left(\frac{p}{M-1}\right)^k \left(\frac{M-2}{M-1}p\right)^{n-j-k}$$

$$+ \frac{1}{2} \sum_{j=0}^{[n/2]} \binom{n}{j} \binom{n-j}{j} (1-p)^j \left(\frac{p}{M-1}\right)^j \left(\frac{M-2}{M-1}p\right)^{n-2j}. \tag{17}$$

If side information is available (i.e., channel monitoring provides information about the presence or absence of interference), instead of Eq. (16), we should use

$$P_j = \bar{P}_M(j) \le (M-1)\,\bar{Q}_M(j)\,. \tag{18}$$

$\bar{P}_M(\cdot)$ is defined in Eq. (14) where $\bar{p}$ is obtained from Eqs. (10) and (11) with $m = 1$, $M = 2$, and $N_s = N_b$ for binary codes and from Eqs. (10) and (11) with $m = 1$, and $M > 2$ for the nonbinary codes. $Q_M(j)$ is the probability of error between two codewords of an M-ary repetition code of rate $1/j$. When side information is available, this is given by

$$\bar{q}_n = \bar{Q}_M(n) = \epsilon_s^n Q_M(n;\bar{p}) + \sum_{l=0}^{n-1} \binom{n}{l} \epsilon_s^l (1-\epsilon_s)^{n-l} Q_M(n-l;P_0), \tag{19}$$

where $Q_M(\cdot;\cdot)$ is defined in Eq. (17) and $\bar{p}$ can be obtained from Eq. (10) by using $m = 1$ and $M > 2$.

3.4 Dual-*k* Convolutional Codes

For dual-k convolutional codes of constraint length k (see Refs. 8 and 14), the alphabet size is 2^k, and codes are used with M-ary FSK modulation ($M = 2^k$). All M-ary FSK symbols within the frequency slot are assumed to be interleaved (the interleaving depth is $N_b/\log_2 M$). The performance of a Viterbi decoder for these codes can be evaluated using the following result from Ref. 14:

$$P_{e,b} \le 2^{k-1} \sum_{j=0}^{\infty} (j+1) \sum_{l=0}^{j} \binom{j}{l} a^l b^{j-l} q_{2v+vj-l}, \tag{20}$$

where $1/v$ (v is a positive integer) is the rate of the code, $a = v$, $b = 2^k - 1 - v$, and q_n is the error probability between two codewords of a repetition code of length n on an M-ary symmetric channel.

In Ref. 14 and more recently in Ref. 6, upper bounds on Eq. (20) are obtained, and they are cited here for reference. The bound in Eq. (21a) (taken from Ref. 6) is tighter than the bound in Eq. (21b)* (taken from Ref. 14):

$$P_{e,b} \le 2^{k-1} \left\{ \sum_{j=0}^{J} (j+1) \sum_{l=0}^{j} \binom{j}{l} a^l b^{j-l} q_{2v+vj-l} + D^{2v} \frac{c^{J+1}[1+(J+1)(1-c)]}{(1-c)^2} \right\} \tag{21a}$$

$$\le \frac{2^{k-1} D^{2v}}{[1 - vD^{v-1} - (2^k - 1 - v)D^v]^2}, \tag{21b}$$

where $c = aD^{v-1} + bD^v < 1$. The quantity D, which is used in Eq. (21), is the Bhattacharyya distance, and it is used in upper-bounding q_j as $q_j \le D^j$.

*This is true for any positive integer J; the tightness of the bound increases with increasing J.

When side information is not available, q_n, the probability of error between two codewords of an M-ary repetition code of rate $1/n$, is given by Eq. (17), and the Bhattacharyya distance is given by Ref. 15 as

$$D = \frac{M-2}{M-1} p_s + 2\sqrt{(1-p_s)p_s/(M-1)}. \tag{22}$$

For our channel model, p_s in Eq. (22) can be obtained from Eq. (2) by using $m = 1$ and $M > 2$.

When side information is available, the majority vote decision rule described in Section 3.3 is used to decide which path of weight n in the trellis the received word is closer. Now, $\bar{q}_n$ is given by Eq. (19), and the Bhattacharyya distance becomes

$$\bar{D} = (1-\epsilon_s)\left[\frac{M-2}{M-1}P_0 + 2\sqrt{(1-P_0)P_0/(M-1)}\right] + \epsilon_s\left[\frac{M-2}{M-1}\bar{p} + 2\sqrt{(1-\bar{p})\bar{p}/(M-1)}\right] \tag{23}$$

where $\bar{p}$ is given by Eq. (10) by using $m = 1$ and $M > 2$.

3.5 Concatenated Codes

In this section, we consider two basic concatenated coding schemes: (1) inner detection-only block codes/outer RS codes, and (2) inner convolutional codes (binary or dual-k)/outer RS codes. In all cases, we assume that there is no channel monitoring, so no side information is available. Bounded distance decoding is used for the outer RS codes.

Inner Detection-Only Block Codes/Outer RS Codes

This concatenation scheme uses the inner code to detect errors within a hop (see Ref. 16 for the case when partial-band jamming is the only form of interference). One codeword of the inner code is used in one RS symbol. When an error is detected, every symbol of the codeword of the inner code (and thus, the corresponding symbol of the RS outer code) is erased. There are, however, errors that are not detected and which result in errors at the output of the inner decoder. The outer code then attempts to correct the errors and erasures of the inner code. We consider two decoding strategies for the outer RS decoder: erasure/error decoding (see Section 3.1, case 2) and parallel erasure/error decoding (see Section 3.1, case 3).

Let n be the block length of the inner code and k the number of information symbols in a codeword. The probability of an undetected error P_{ud} (i.e., when a nonzero error pattern satisfies all of the parity-check equations) is given by Ref. 12:

$$P_{ud} = \sum_{i=1}^{n} A_i \, \alpha_{n,i}, \tag{24}$$

where A_i is the number of codewords with Hamming weight i and $\alpha_{n,i}$ is the probability that in a sequence of n received symbols a particular pattern of i symbol errors occur. Furthermore, the probability of a detected error P_d is given by Ref. 12 as

$$P_d = 1 - \alpha_{n,0} - P_{ud}, \tag{25}$$

since $1 - \alpha_{n,0}$ is the probability of at least one symbol error. For our system and channel model, $\alpha_{n,i}$ is upper-bounded by

$$\alpha_{n,i} < \begin{cases} (1-P_h)^{K-1}\left[(1-\rho)\left[\dfrac{P_0}{M-1}\right]^i (1-P_0)^{n-i} + \rho\left[\dfrac{P_{J,0}}{M-1}\right]^i (1-P_{J,0})^{n-i}\right] & ;0 \le i < n \\ (1-P_h)^{K-1}\left[(1-\rho)\left[\dfrac{P_0}{M-1}\right]^n + \rho\left[\dfrac{P_{J,0}}{M-1}\right]^n\right] + \dfrac{1-(1-P_h)^{K-1}}{(M-1)^n} & ;i = n, \end{cases} \tag{26}$$

where P_0 and $P_{J,0}$ are the conditional error probabilities of an M-ary FSK system given that thermal noise or jammer plus thermal noise is present (P_0, $P_{J,0}$ were defined in Section 3.1).

For high rate codes ($r = k/n$ close to 1), the A_i, $0 \le i \le n$, are easily calculated by using the McWilliams identities (since a high rate code has more codewords than its dual). If $A(z)$ is the weight enumerator of the (n,k) code and $B(z)$ (B_i, $0 \le i \le n$) is that of its dual, then as in Ref. 17:

$$A(z) = M^{-(n-k)}[1 + (M - 1)z]^n B\left[\frac{1 - z}{1 + (M - 1)z}\right]. \tag{27}$$

For codes with a single parity-check symbol, $B_0 = 1$, $B_n = M - 1$, and $k = n - 1$, so that $B(z) = 1 + (M - 1)z^n$, and we obtain

$$A_n = [(M - 1)^n + (-1)^n(M - 1)]/M \tag{28a}$$

and

$$P_{ud} = (1 - P_h)^{K-1}\Bigg[\rho\left\{\frac{1}{M}\left[1 + (M - 1)\left[1 - \frac{MP_{J,0}}{M - 1}\right]^n\right] - (1 - P_{J,0})^n\right\}$$
$$+ (1 - \rho)\left\{\frac{1}{M}\left[1 + (M - 1)\left[1 - \frac{MP_0}{M - 1}\right]^n\right] - (1 - P_0)^n\right\}\Bigg]$$
$$+ \frac{1 - (1 - P_h)^{K-1}}{(M - 1)^n} A_n. \tag{28b}$$

To find the error probability at the output of the outer (RS) decoder in the case of erasure/error decoding, we need only substitute P_{ud} from Eq. (28) for p_s and P_d from Eq. (25) for ϵ_s in Eq. (7).

In the case of parallel erasure/error decoding, the outer decoder attempts to correct both erasures (the detected errors of the inner decoder) and errors (the undetected errors of the inner decoder) when the number of the erasures is less than $\tilde{e} = \tilde{n} - \tilde{k}$ (the erasure correction capability of the outer RS $(\tilde{n},\tilde{k})$ code); if the number of erasures is larger than $\tilde{e}$, the RS decoder switches to the error-correction mode.

To evaluate the probability of a symbol error at the output of the outer decoder for the erasure/error-correction mode (denoted by $P_{e,s;1}$), we need only substitute P_d from Eq. (25) and P_{ud} from Eq. (28) for e_s and p_s of Eq. (7), respectively, and replace n and e of Eq. (7) with $\tilde{n}$ and $\tilde{e}$.

To evaluate the probability of a decoder symbol error for the error-correction mode (denoted by $P_{e,s;2}$), we observe that, given that more than $\tilde{e}$ errors were detected by the inner decoder, a codeword error at the outer decoder occurs when the number of detected and undetected errors is greater than $\tilde{t} = \lfloor(\tilde{n} - \tilde{k})/2\rfloor$ (the error-correction capability of the outer RS code). Since the detected and undetected error events are mutually exclusive, we obtain

$$P_{e,s;2} = \sum_{j=\tilde{e}+1}^{\tilde{n}} \binom{\tilde{n}}{j} P_d^j (1 - P_d)^{\tilde{n}-j} \sum_{0 \le l \le \tilde{n}-j} \frac{j+l}{\tilde{n}} \binom{\tilde{n}-j}{l} P_{ud}^l (1 - P_{ud})^{\tilde{n}-j-l}, \tag{29}$$

and, finally, the overall symbol error probability is $P_{e,s} = P_{e,s;1} + P_{e,s;2}$.

Inner Convolutional Code/Outer RS Code

In this case, the binary or dual-k convolutional inner codes use binary or M-ary ($M = 2^k$) FSK modulation, respectively, with noncoherent demodulation, hard decisions, and Viterbi decoding. To evaluate the performance of these two coding schemes when they are concatenated with RS outer codes, we proceed in the following way: Let P_e be the prespecified value of the tolerable error probability. Let $(\tilde{n}, \tilde{k})$ be the parameters of the outer RS code. Using the error-correction capability $\tilde{t} = \lfloor (\tilde{n} - \tilde{k})/2 \rfloor$ of the RS code and P_e, one can determine the maximum outer symbol error probability p_s (for error-only decoding) that yields a BER smaller than or equal to P_e [either from existing tables or from Eqs. (4) and (2)]. Once p_s has been obtained, the expressions for binary or dual-k CC (found in Sections 3.3 and 3.4, respectively) can be used to find $(E_b/N_J)_{\min}$ for given K or $K_{\max}$ for given E_b/N_J.

3.6 Reed-Solomon Codes with Diversity

In this section, we consider the coupling of diversity of order L with Reed-Solomon (n,k) codes. Each M-ary symbol is transmitted L times; each RS symbol contains m M-ary symbols. All three cases examined in Section 3.1 are also examined here. In all cases, it is assumed that the L diversity transmissions take place through independent channels (e.g., during different dwell times) as a result of the use of interleaving. The FH/SS system with diversity is thus equivalent to a fast-frequency-hopping system. Bounded distance decoding of the RS code is employed.

For case (1), error-only decoding without information about the state of the channel available, majority vote combining of the L diversity transmissions is used. The bit error probability can be upper-bounded by Eq. (4), in which p_s should be replaced by

$$p_s(L) = 1 - [1 - P_M(L;p)]^m, \tag{30}$$

where $P_M(\cdot;\cdot)$ is defined in Eq. (13), and p can be obtained from p_s of Eq. (2) for $m = 1$.

For case (2), erasure/error decoding with perfect information about the state of the channel available, the decoder erases a symbol of the RS code if and only if all L diversity transmissions of at least one out of the m M-ary symbols within the RS symbol are hit by interference. Otherwise, the decoder attempts to correct errors and uses majority vote decoding on the diversity transmissions that were not hit by interference for each M-ary symbol within the RS symbol. The bit error probability is now upper-bounded by Eq. (7), where ϵ_s should be replaced by

$$\epsilon_s(L) = 1 - (1 - \epsilon_s^L)^m, \tag{31}$$

in which ϵ_s can be obtained from Eq. (5) by setting $m = 1$ and $M > 2$, and p_s should be replaced by

$$\bar{p}_{s,0}(L) = 1 - \left[1 - \sum_{l=0}^{L-1} \binom{L}{l} \epsilon_s^l (1 - \epsilon_s)^{L-l} P_M(L - l; P_0) \right]^m, \tag{32}$$

where $P_M(\cdot;\cdot)$ can be obtained from Eq. (12) for $M = 2$ and Eq. (13) for $M > 2$.

Finally, for case (3), parallel erasure/error decoding with channel information available, the decoder erases symbols according to the same rule as for case (2) and if the number or erased symbols is smaller than $e = n - k$, it uses majority vote decoding in an identical way. However, if the number of erased symbols becomes larger than e, the decoder switches to the error-correcting mode. It uses majority vote decoding based on all L diversity transmissions of a particular M-ary symbol when all L transmissions are hit by interference and majority vote decoding based on the diversity transmissions that are not hit by interference when fewer than L transmissions are hit. This scheme is an extension of the scheme of Ref. 5 to our channel model and is optimal in the absence of AWGN or multiuser interference, as shown in Ref. 6. The decoded symbol error probability is upper-bounded by $P_{e,s} = P_{e,s;1} + P_{e,s;2}$ (and thus, the bit error probability can also be upper-bounded by this quantity), where

$$P_{e,s;1} = \sum_{j=0}^{e} \binom{n}{j} (\epsilon_s^L)^j (1 - \epsilon_s^L)^{n-j} \sum_{\substack{e+1 \leq 2l+j \\ l+j \leq n}} \frac{l}{n} \binom{n-j}{l} \bar{p}_{s,0}(L)^l [1 - \bar{p}_{s,0}(L)]^{n-l}, \tag{33}$$

in which $\bar{p}_{s,0}$ is given by Eq. (32), and

$$P_{e,s;2} = \sum_{j=e+1}^{n} \binom{n}{j} (\epsilon_s^L)^j (1 - \epsilon_s^L)^{n-j} \sum_{\substack{t+1 \leq l_1 + l_2 \\ l_1 \leq j \\ l_2 \leq n-j}} \frac{l_1 + l_2}{n} \binom{j}{l_1} \bar{p}_s(L)^{l_1} [1 - \bar{p}_s(L)]^{j-l_1}$$

$$\binom{n-j}{l_2} \bar{p}_{s,0}(L)^{l_2} [1 - \bar{p}_{s,0}(L)]^{n-j-l_2}, \tag{34}$$

in which $\bar{p}_s(L)$ is given by

$$\bar{p}_s(L) = 1 - [1 - P_M(L;\bar{p})]^m, \tag{35}$$

where $P_M(\cdot;\cdot)$ is defined by Eq. (12) and Eq. (13) for $M = 2$ and $M > 2$, respectively, and $\bar{p}$ can be obtained from Eq. (10) by setting $m = 1$ and $M > 2$.

4. NUMERICAL RESULTS

The numerical results presented in this section are separated into two groups. The first group provides the minimum signal (actually, information bit)-to-jammer energy ratio (E_b/N_J) required to achieve a desirable bit error probability of P_e (typically 10^{-3}) for a variety of error-control coding schemes. E_b/N_J is given as a function of the percentage of the band jammed (ρ), when K—the total number of users transmitting in the vicinity of a particular receiver—and the other channel parameters are fixed. The second group provides the maximum number of transmitting users ($K_{\max}$) that can be tolerated in the vicinity of a receiver (including the desired signal), so that the bit error probability is below a desirable level P_e for a variety of error-control coding schemes. $K_{\max}$ is given as a function of ρ, when the signal-to-jammer power ratio (E_b/N_J) and the other system and channel parameters are held fixed.

In both cases, q denotes the number of frequencies used for frequency-hopping, N_b the number of bits per dwell time, E_b/N_0 the signal-to-AWGN ratio, M the number of orthogonal signals (or the frequency tones) used for the M-ary FSK modulation (with noncoherent demodulation), and m the number of M-ary symbols in each symbol of the Reed-Solomon code. All the results that follow are concerned with asynchronous FH/SSMA systems.

The presentation of results from the first group starts with Fig. 1, where E_b/N_J vs ρ is plotted for a Reed-Solomon RS(64,32) code (rate 1/2) over the Galois field $GF(8^2)$ (i.e., 8-ary FSK signaling is used, and each RS 64-ary symbol consists of two 8-ary symbols) with either error-only or parallel erasure/error decoding. The values of the system and channel parameters are $P_e = 10^{-3}$, $K = 5$, $E_b/N_0 = 12$ dB, and the relative power of the faded component of the Rician channel γ^2 is varying [$\gamma^2 = 0$ (AWGN), .01, .1, 1, and 10]. For the case of error-only decoding and $\gamma^2 = 1$ or 10, a bit error probability smaller than or equal to 10^{-3} cannot be achieved with this code rate (1/2); this is why these two curves of E_b/N_J vs ρ are missing. As we will see later, a lower code rate can overcome this problem.

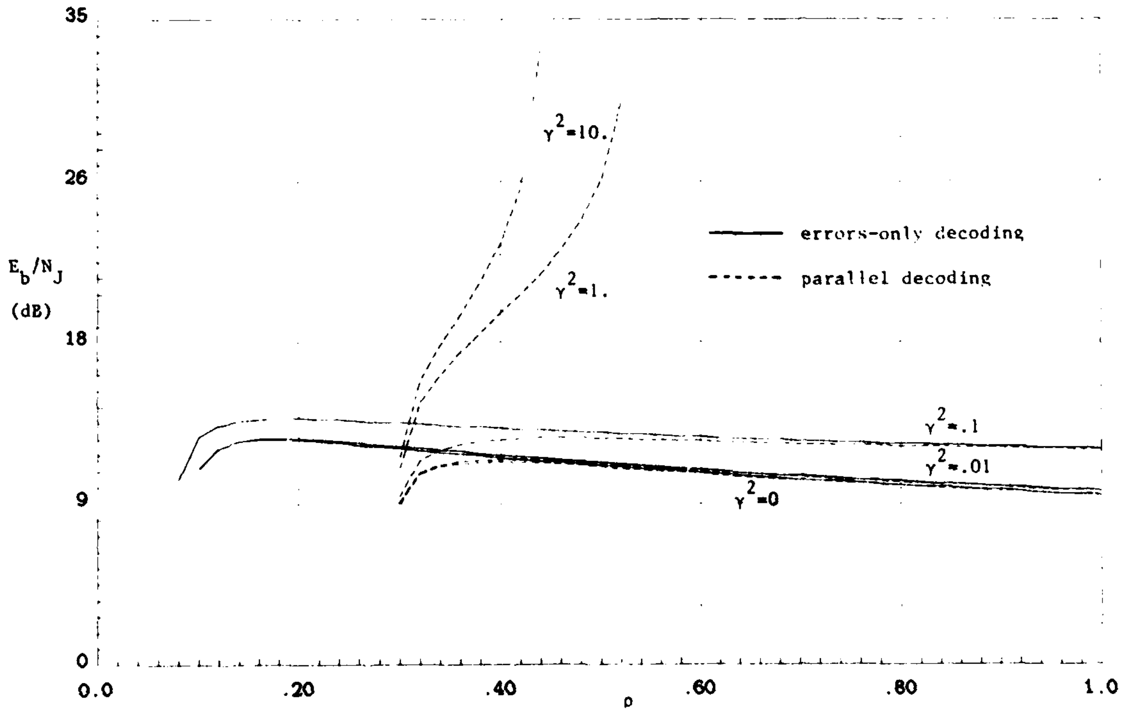


Fig. 1 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using $RS(64,32)$ codes with error-only and parallel erasure/error decoding ($q = 100$, $N_b = 12$, $M = 8$, $m = 2$, $E_b/N_0 = 12$ dB, $K = 5$); Rician fading channel with varying γ^2.

In the same figure, notice the improvement that the parallel erasure/error decoding scheme offers over the error-only decoding scheme for both $(E_b/N_J)_{\max}$ (the maximum E_b/N_J required to achieve a bit error probability of $P_e = 10^{-3}$ as ρ varies between 0 and 1) and ρ^*. The quantity ρ^* is defined as the maximum value of ρ for which the specified performance is achieved independently of the power of the jammer. These two parameters are important when characterizing the performance of the FH/SS system against partial-band noise jamming [5,6]. Obviously, it is desirable to decrease the value of $(E_b/N_J)_{\max}$ and to increase the value of ρ^*. The improvement of $(E_b/N_J)_{\max}$ is more substantial for larger values of γ^2 (e.g., $\gamma^2 = 1.$ and 10). The improvement of ρ^* is about .2 (from .08 to .28). Parallel decoding of RS codes is superior to error-only decoding and should be preferred for combatting severe interference. Regarding the deterioration of the receiver performance as γ^2 increases from 0 (AWGN) to .01 and .1, notice that the increase in E_b/N_J is modest (it is .3 dB and 1.9 dB, respectively, at $\rho = .4$), but as γ^2 becomes 1. and 10., the increase in the required E_b/N_J becomes substantial.

In Fig. 2, E_b/N_J is plotted vs ρ for a RS(32,8) code (rate 1/4) with parallel erasure/error decoding and the cases ($M = 2$, $m = 5$),($M = 32$, $m = 1$) for $K = 5$ and a Rician fading channel

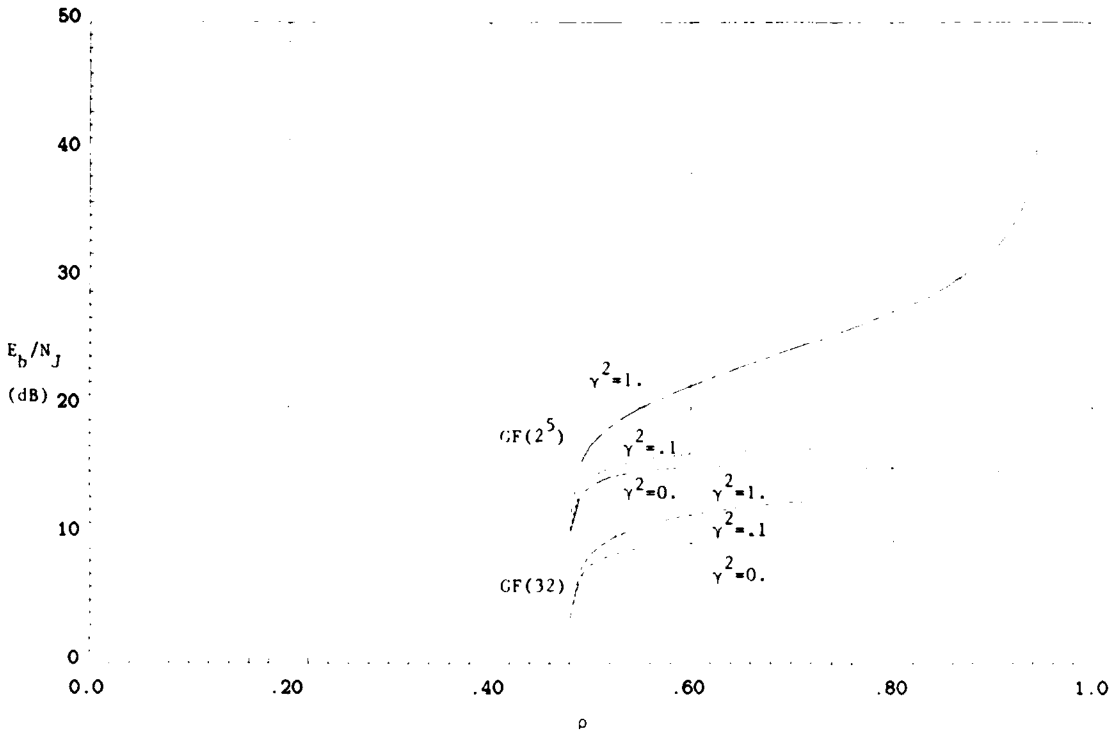


Fig. 2 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using $RS(32,8)$ codes with parallel erasure/error decoding over various $GF(M^m)$, with corresponding M-ary FSK modulation ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); Rician fading channel with varying γ^2.

with varying γ^2. Comparing the two cases shows that ρ^* does not change considerably but that $(E_b/N_J)_{\max}$ does: from 8.8 dB for $M = 32$ to 15.6 dB for $M = 2$ for the AWGN channel ($\gamma^2 = 0$). As the relative power in the fading component increases from $\gamma^2 = 0.$ to .1, and then to 1., the difference in the required $(E_b/N_J)_{\max}$ increases substantially between the two schemes; the scheme with $M = 32$ requires substantially less signal-to-jammer power ratio to achieve the same bit error probability. Therefore, it is preferable to use M-ary instead of binary FSK modulation in this case.

Figure 3 shows similar results for an RS(64,16) code (rate 1/4), the same systems parameters, and for the cases ($M = 2$, $m = 6$), ($M = 4$, $m = 3$), and ($M = 8$, $m = 2$). Similar observations as for Fig. 2 can be made.

Figure 4 shows E_b/N_J plotted vs ρ for repetition codes of rates $1/L$ = 1/4, 1/5, 1/6, 1/7, 1/8, and 1/9 without side information and with 32-ary FSK modulation. For the parameters given in the figure caption, it appears that the rate 1/8 repetition code is the optimal code. For rates smaller than 1/8, the principle of diminishing returns manifests itself, due to the noncoherent combining loss.

Figure 5 shows the performance of the same repetition codes as in Fig. 4 but with side information, for the same system parameters. Notice that now the rate 1/8 code is optimal for $(E_b/N_J)_{\max}$ but not for ρ^* (ρ^* increases as the rate decreases). Comparing $(E_b/N_J)_{\max}$ and ρ^* for Figs. 4 and 5 shows that the former decreases slightly while the latter increases substantially. This results in improvement in both directions when side information is available.

Figure 6 shows the performance of binary convolutional codes with and without side information. Codes of constraint length 9 and rates 1/2 and 1/3 are being considered. The figure reveals the substantial improvement in ρ^* and E_b/N_J for fixed ρ that is provided by reducing the rate of the code and using side information. In the presence of fading ($\gamma^2 = .5$), the performance deteriorates

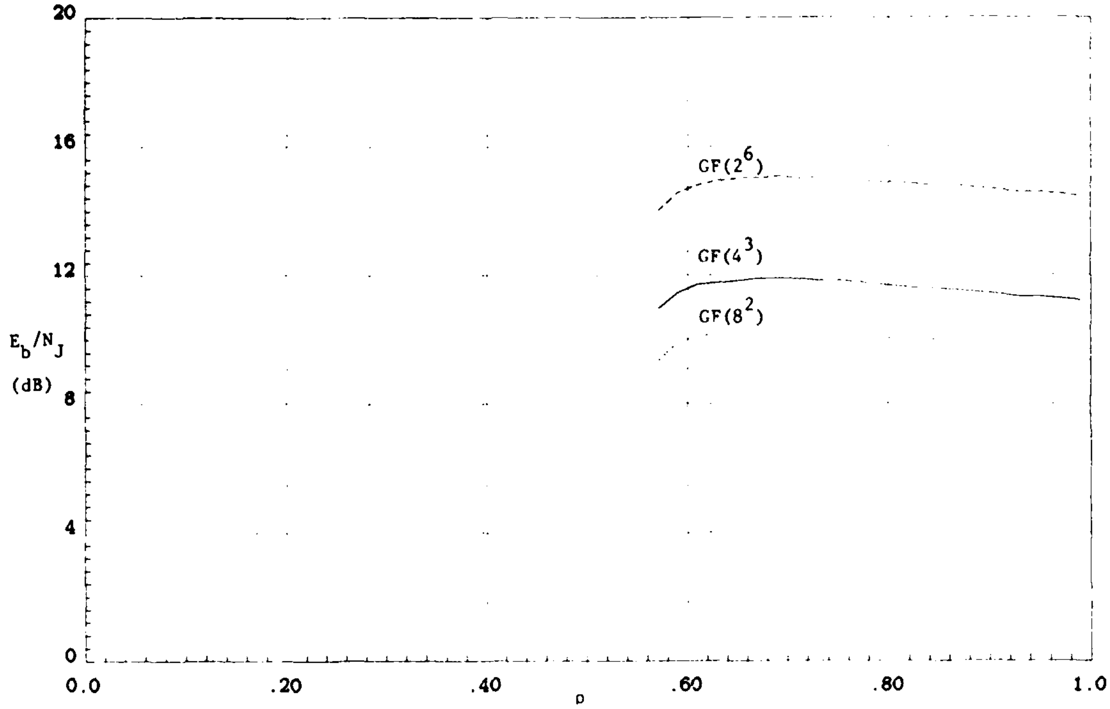


Fig. 3 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using $RS(64,16)$ codes with parallel erasure/error decoding over various $GF(M^m)$, with corresponding M-ary FSK modulation ($q = 100$, $N_b = 12$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel.

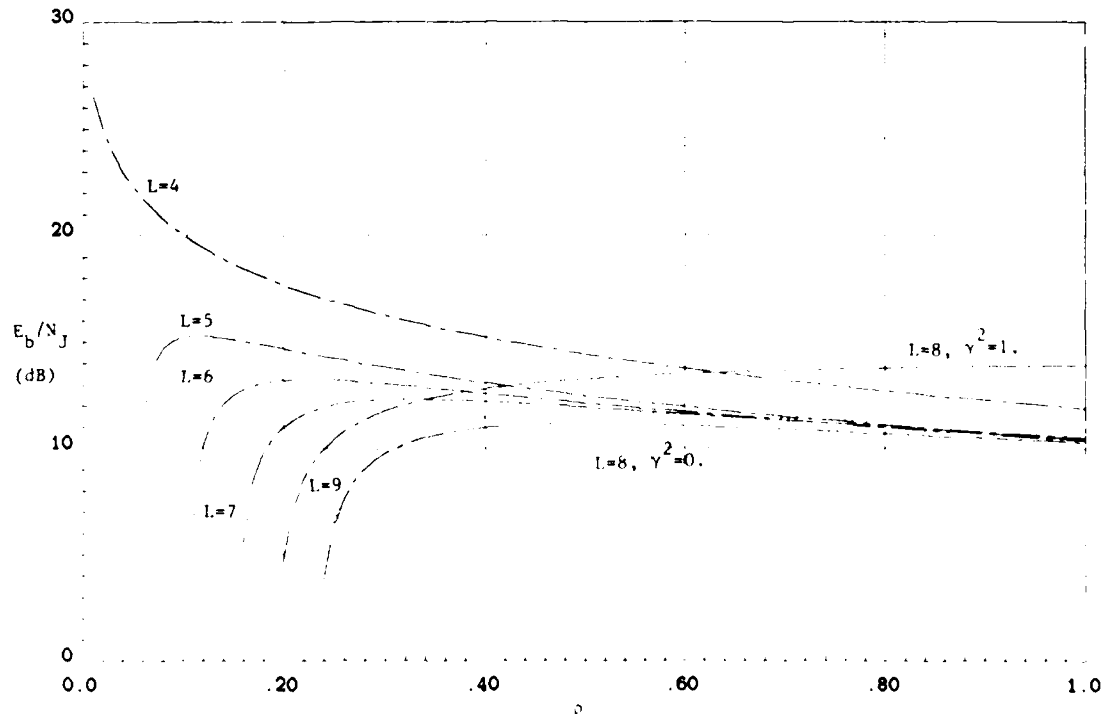


Fig. 4 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK and varying diversity without side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel; for $L = 8$, AWGN channel, and Rician fading channel with $\gamma^2 = 1$.

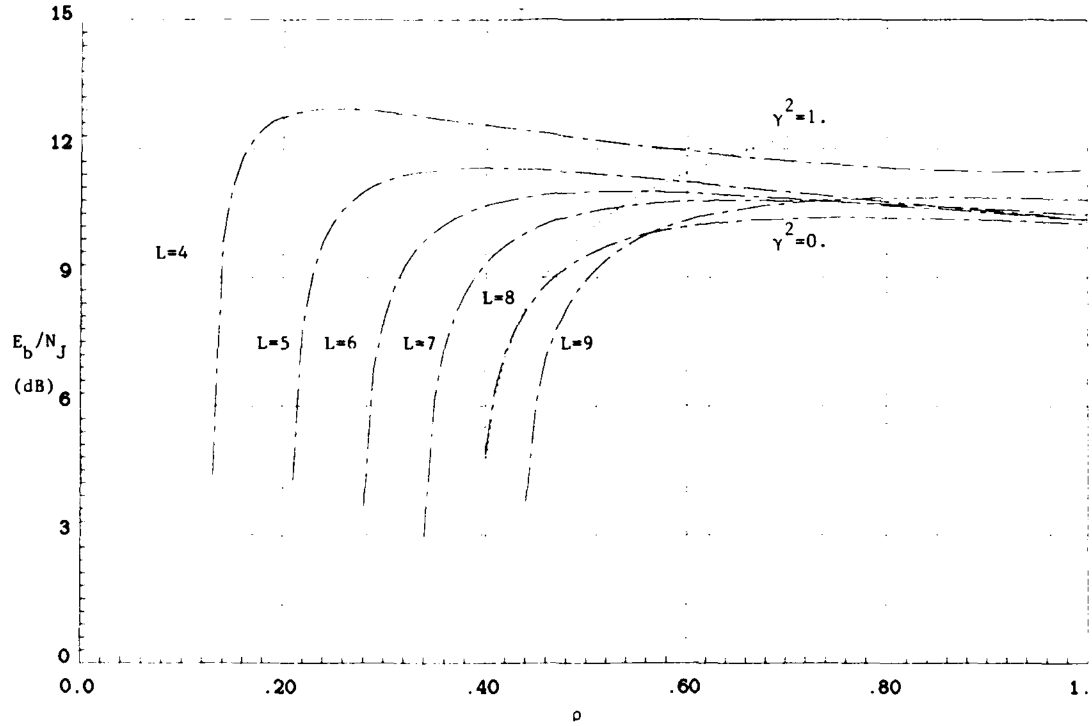


Fig. 5 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK and varying diversity with side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel; for $L = 8$, AWGN channel, and Rician fading channel with $\gamma^2 = 1$.

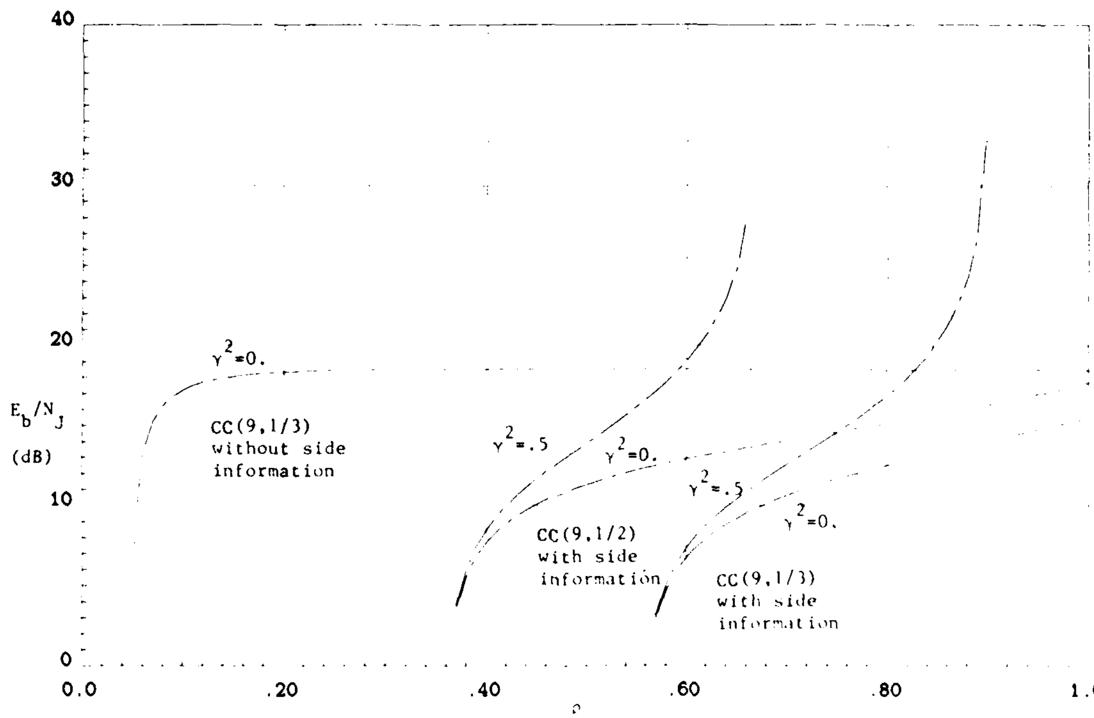


Fig. 6 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using binary convolutional codes of constraint length 9 and code rates 1/2 and 1/3, with and without side information; ($q = 100$, $N_h = 12$, $E_b/N_0 = 12$ dB, $K = 5$); AWGN channel, and Rician fading channel with $\gamma^2 = .5$.

considerably with respect to E_b/N_J. The missing curves for the cases CC(9,1/2) without side information and $\gamma^2 = 0.$ or .5 and CC(9,1/3) without side information and $\gamma^2 = .5$ show that a bit error probability of 10^{-3} cannot be achieved by these codes under the specified channel conditions.

Figure 7 shows the performance of nonbinary convolutional codes of constraint length 7 and code rates of 1/2 (information symbols per code symbol) with 4-ary FSK modulation and 1/3 (information symbols per code symbol) with M = 8-ary FSK modulation (i.e., the effective code rate for both codes is 1 bit per code symbol). For these results, we use the union bound cited in Eqs. (17) and (19) for the cases of no side information and side information, respectively, to compute P_j in Eq. (15). The code that uses 8-ary FSK modulation performs better than the one that uses 4-ary FSK modulation for all the cases considered. The use of side information improves the performance of the codes considerably. The CC(7,1/2) $M = 4$ code can not achieve a bit error probability of 10^{-3} for a fading channel with $\gamma^2 = .5$.

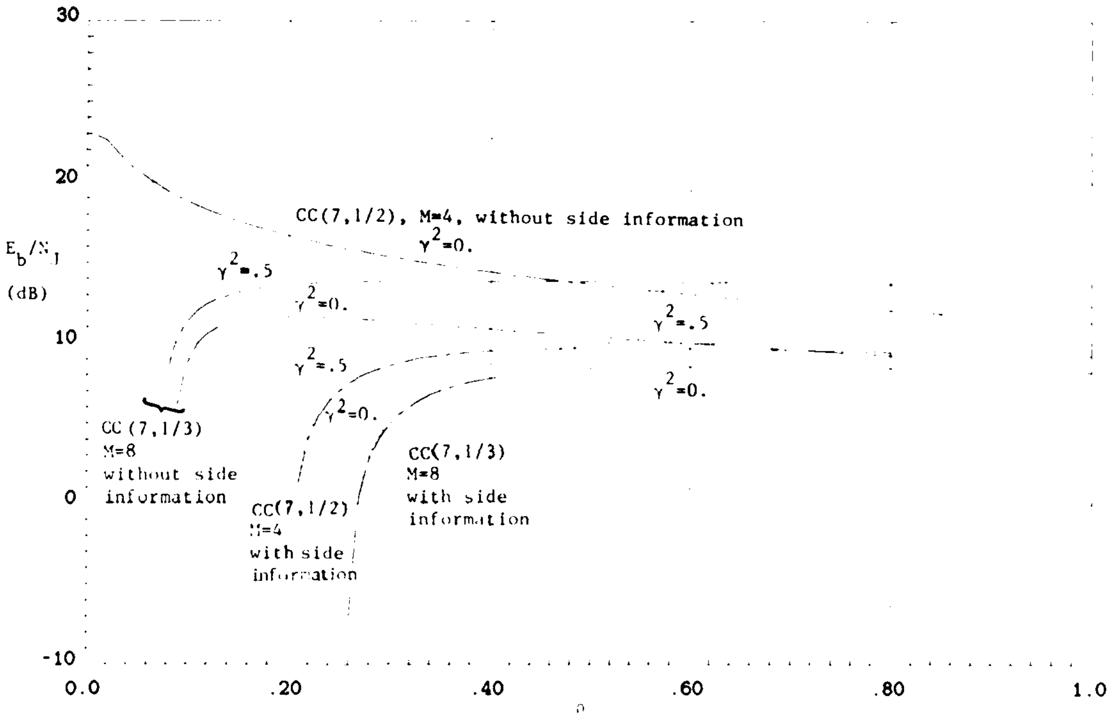


Fig. 7 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using nonbinary convolutional codes of constraint length 7, and code rates 1/2 ($M = 4$) and 1/3 ($M = 8$) with or without side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel, and Rician fading channel with $\gamma^2 = 5$.

Figure 8 shows the performance of dual-5 convolutional codes both with and without side information; 32-ary FSK modulation and the upper bound of Eq. (21a) with $J = 5$ terms are used. Notice the increase in ρ^* for the case without side information. Also notice that, in the case of side information, E_b/N_J becomes unbounded for $\rho > .55$. This is caused by the expressions of Eq. (21) that are used to upperbound the performance of the coded FH/SS system: as ρ increases, c in Eq. (21a) approaches 1, and the bound becomes arbitrarily large; this is a deficiency of the available bounds but does not imply that the actual performance follows this pattern of behavior.

Figure 9 shows the performance of Reed-Solomon codes with varying diversity; 32-ary FSK modulation and parallel erasure/error decoding are used. Increasing the diversity causes a substantial improvement to the value ρ^* and a slight deterioration to the value of $(E_b/N_J)_{max}$. Of course, this is provided at the expense of a lower overall code rate and subsequently of a larger bandwidth expansion.

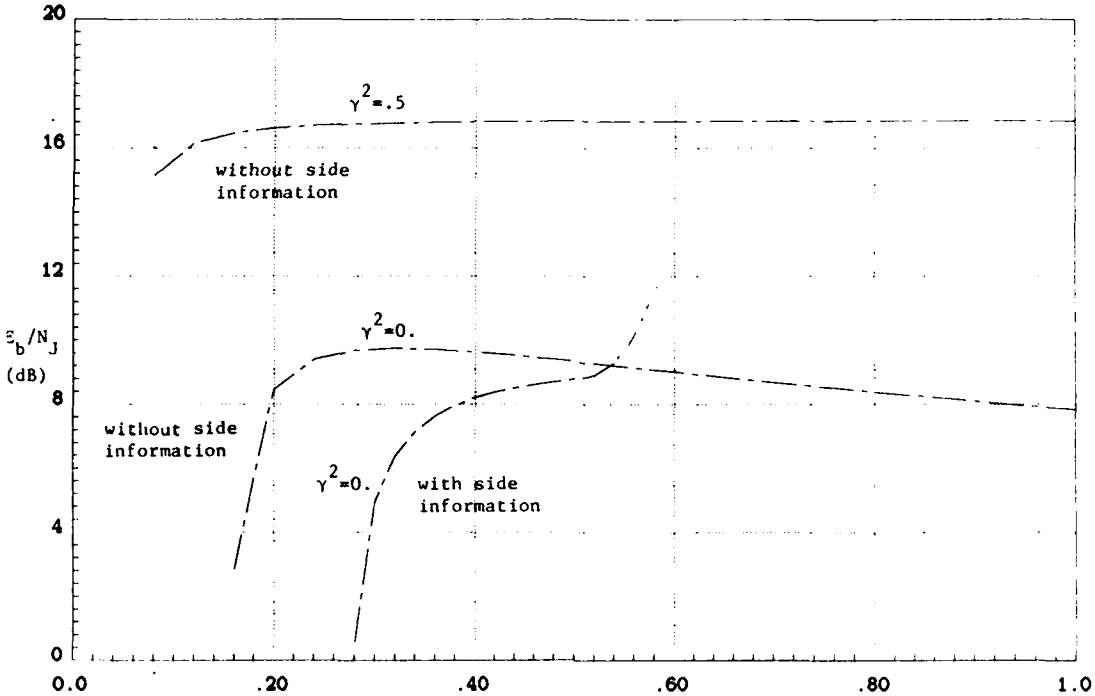


Fig. 8 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using a dual convolutional code of rate 1/4, with or without side information ($q = 100$, $N_b = 15$, $M = 32$, $E_b/N_0 = 12$ dB, $K = 5$); AWGN channel, and Rician fading channel with $\gamma^2 = .5$.

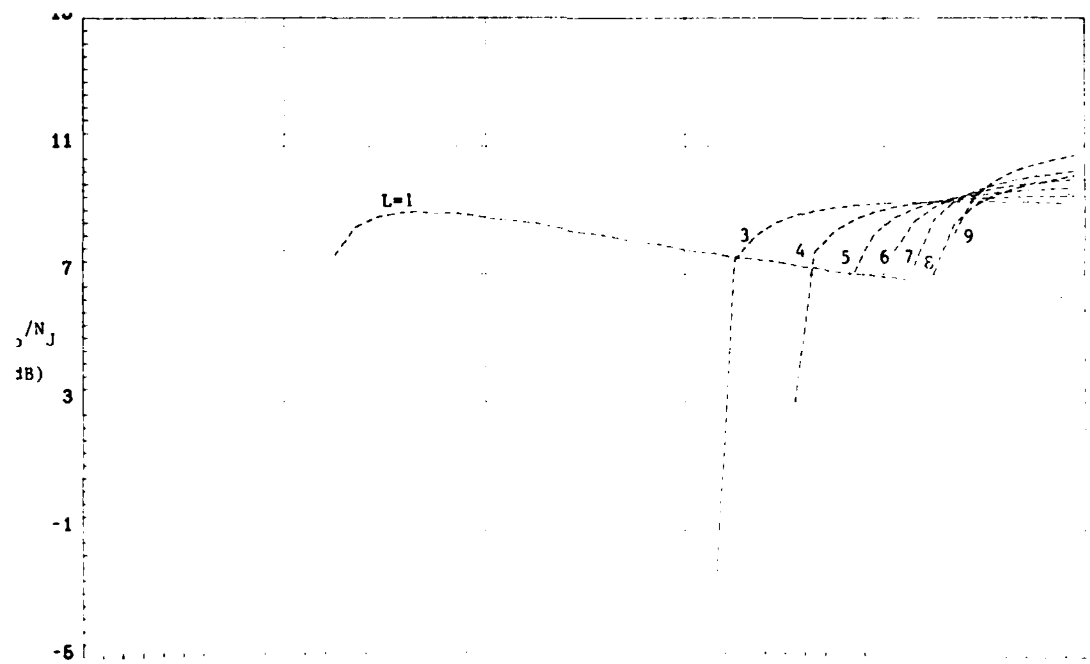


Fig. 9 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK with $RS(32,16)$ codes and parallel erasure/error decoding plus varying diversity ($q = 100$, $N_h = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel.

Figure 10 compares the performance of repetition codes and Reed-Solomon codes of rate 1/4 that use 32-ary FSK modulation. Repetition codes with or without side information and Reed-Solomon codes with error-only or parallel error/erasure decoding are considered. The superiority of Reed-Solomon codes over repetition codes and of parallel decoding over error-only decoding is clear for both performance measures, $(E_b/N_J)_{\max}$ and ρ^*. Notice that the repetition code with $n = 4$ and without side information cannot achieve a bit error probability of 10^{-3} for fading with $\gamma^2 = .5$.

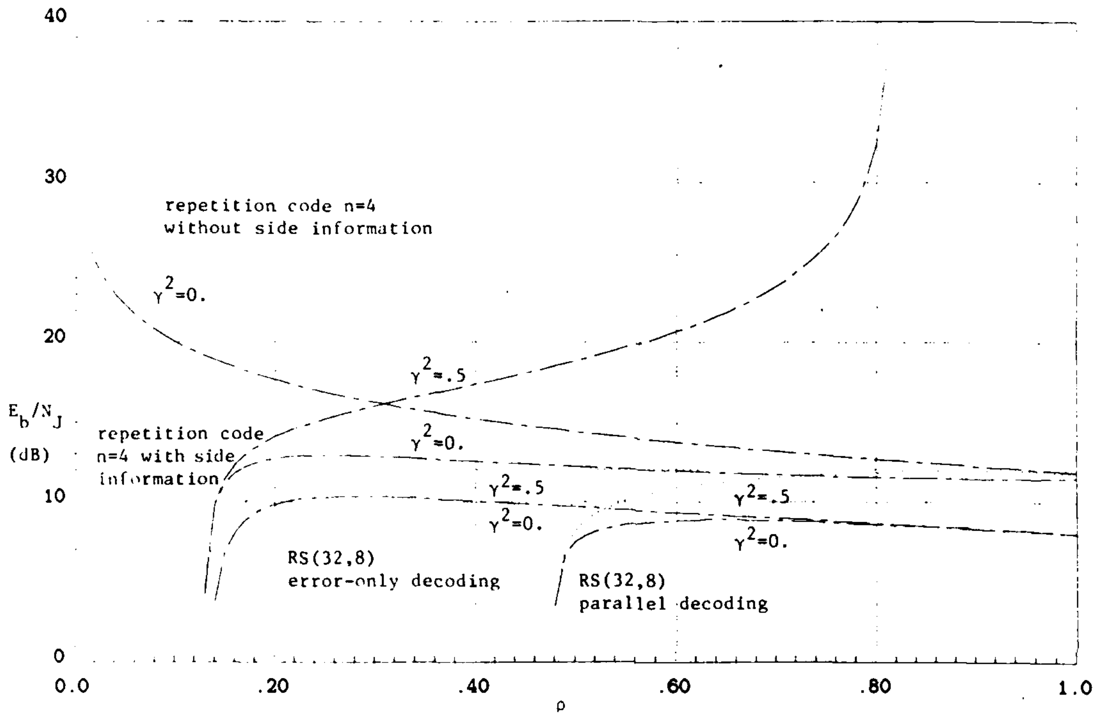


Fig. 10 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK with various rate 1/4 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel, and Rician fading channel for $\gamma^2 = .5$.

Figure 11 shows the performance of several error-control coding schemes of rate 1/8. Repetition codes (with and without side information), Reed-Solomon codes (with error-only or parallel decoding), and Reed-Solomon codes with diversity are considered. For this coding rate (1/8) and in the absence of side information, the RS(32,4) code performs worse than the repetition code with $L = 8$. This is in contrast to the comparison of the rate 1/4 codes presented in Fig. 10. The latter performs worse than the RS(32,16) code with diversity 4 (error-only decoding). However, when side information is available, the RS(32,16) code with diversity 4 and parallel decoding outperforms the RS(32,4) code with parallel decoding and the repetition code with $L = 8$ in both performance measures [ρ^* and $(E_b/N_J)_{\max}$]. Between these two, the former outperforms the latter in ρ^* but not in $(E_b/N_J)_{\max}$.

Figure 12 shows the performance of asynchronous FH/SSMA systems with 32-ary FSK, noncoherent demodulation, and RS(32,8) coding with error-only or parallel decoding for various combinations of q (number of frequency slots) and N_b (number of bits per dwell time) and an AWGN channel. As q increases from 100 to 1000, the jammer is assumed to maintain the same N_J (thus, its total power P_J increases by a factor of 10 and 100, respectively), and all other system and channel parameters are held fixed. Under these conditions, there is a slight increase in ρ^* ($\sim .08$) and a more considerable decrease in $(E_b/N_J)_{\max}$ (~ 2.8 dB) for the case of error-only decoding. For the case of parallel decoding, both performance measures improve as q increases but this is more modest. Furthermore, if q is held fixed and N_b increases from 5 to 12, then there is again some improvement in

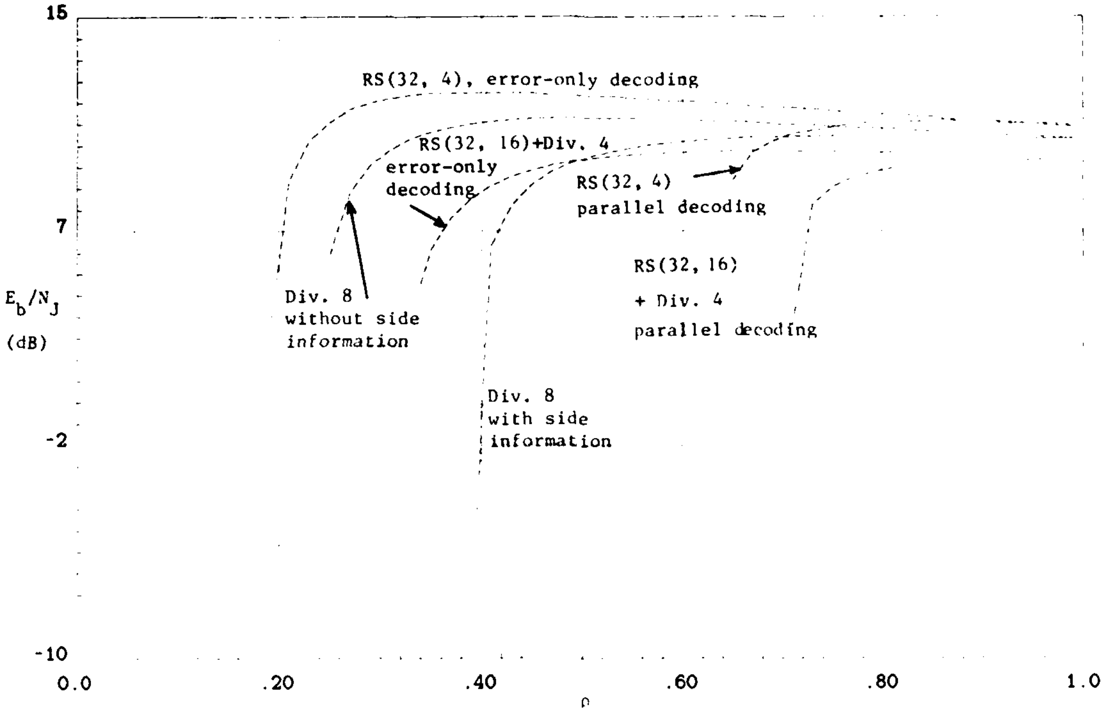


Fig. 11 — Minimum E_b/N_J required for $P_e = 10^{-3}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK with various rate 1/8 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel.

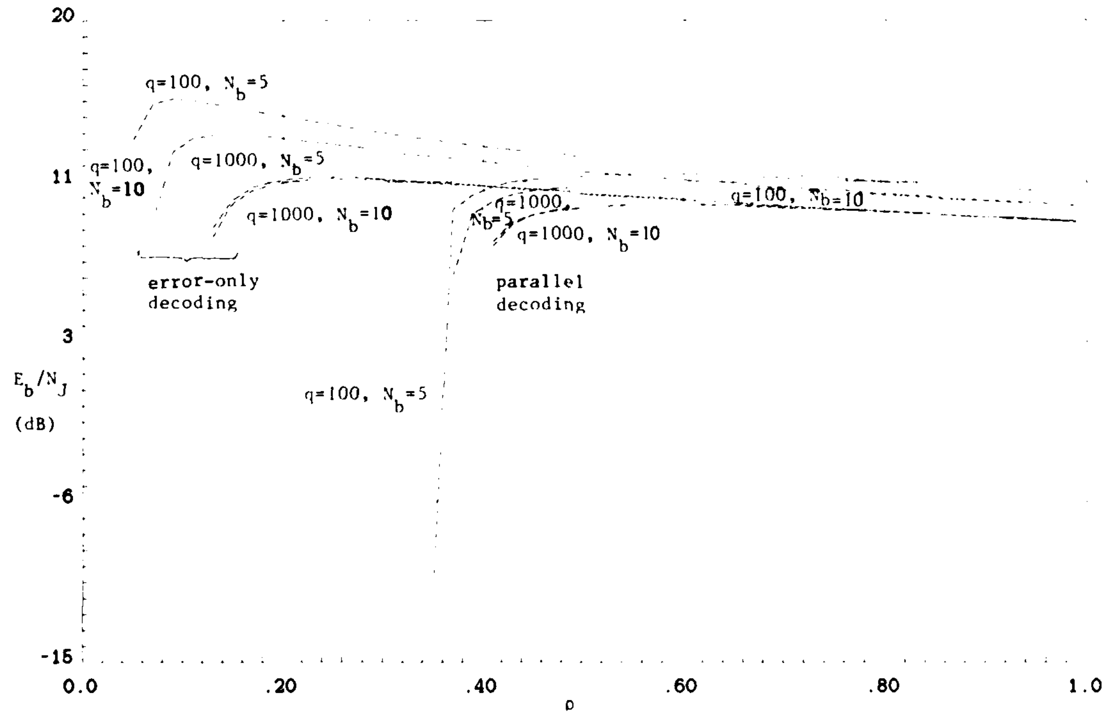


Fig. 12 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK with $RS(32,8)$ codes and various combinations of q and N_b ($E_b/N_0 = 15$ dB, $K = 5$); AWGN channel.

both performance measures. The improvement is more considerable for small q than for larger q and for the case of error-only decoding than for the case of parallel decoding. Usually, the values of q and N_b are determined by considerations other than combatting channel interference; in any case, it is desirable to use a large q and an N_b larger than 1, at least when the interference is expected to be of the form considered in this paper.

Figure 13 shows the performance of a concatenated RS(64,38) outer code with a parity-check (PC) (6,5) inner block code (overall rate approximately 1/2). Both erasure/error decoding and parallel decoding are considered (see Section 3.4). For the system and channel parameters shown in the figure caption, there is very little difference in the performance of the two decoding schemes.

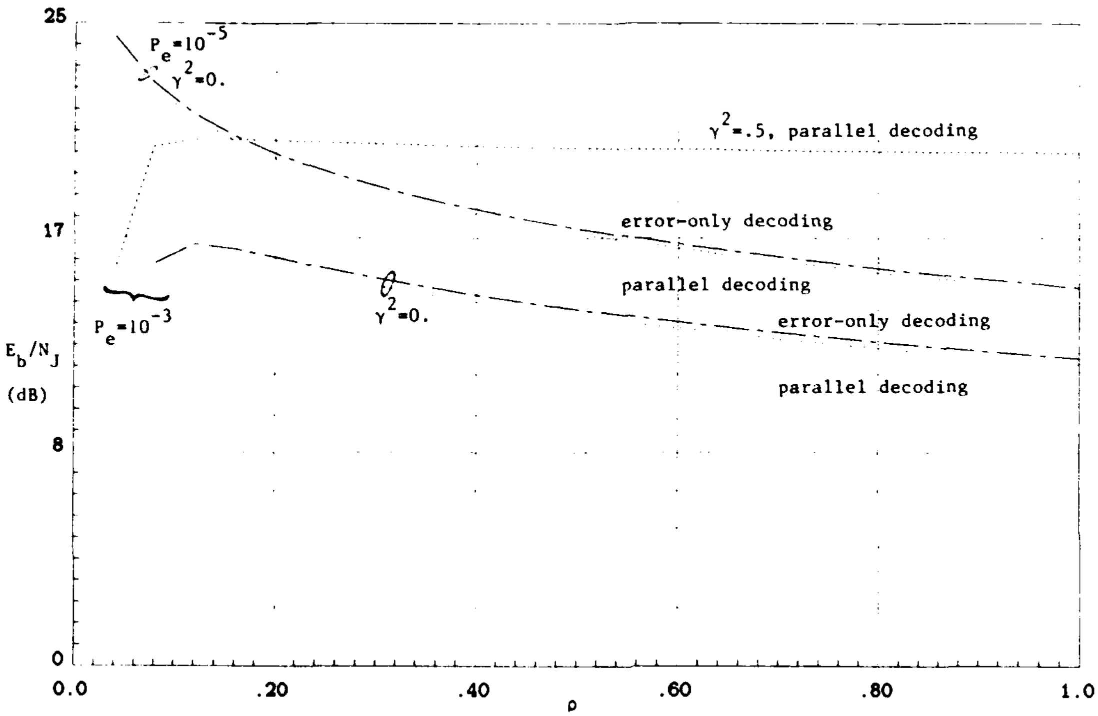


Fig. 13 — Minimum E_b/N_J required for $P_e = 10^{-3}$ and $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using binary FSK with concatenated coding schemes (RS(64,38) error-only/parallel decoding + PC(6,5)) (q = 100, N_b = 12, E_b/N_0 = 20 dB, K = 5); AWGN channel, and Rician fading channel with γ^2 = .5.

Figure 14 compares the performance of RS codes and concatenated codes that use RS outer codes and PC inner codes in the absence of side information. The RS(64,16) code uses error-only decoding; the RS(64,19) + PC(6,5) concatenated code with overall code rate approximately 1/4 uses either erasure/error decoding or parallel decoding. The comparison shows that the concatenation of RS outer codes with inner detection-only (here parity-check) codes substantially improves the system performance with respect to $(E_b/N_J)_{\max}$ (the maximum required signal-to-jammer power ratio to achieve a bit error probability of 10^{-5} when the jammer's ρ varies between 0 and 1)—up to 2.5 dB for $\gamma^2 = 0$, and improves it only modestly with respect to ρ^*. The improvement is more considerable when the parallel decoding scheme is employed.

Figures 15 and 16 show the performance of concatenated RS outer codes with inner binary convolutional codes and dual-k convolutional codes, respectively. In particular, Fig. 15 compares the performance of the RS(32,8) code with that of the concatenated RS(32,16) + CC(9,1/2) code, with overall code rate 1/4, when error-only decoding and binary FSK modulation are used. The concatenated code provides considerable improvement in $(E_b/N_J)_{\max}$ and negligible improvement in ρ^*.

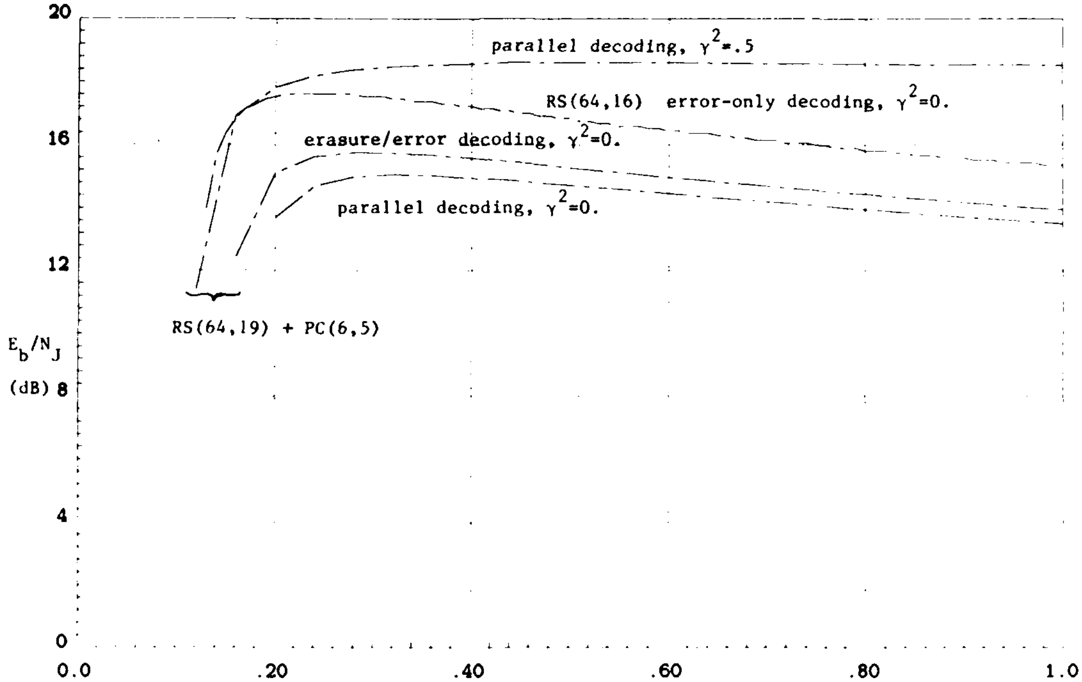


Fig. 14 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using binary FSK with various rate 1/4 coding schemes ($q = 100$, $N_b = 12$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel, and Rician fading channel with $\gamma^2 = .5$.

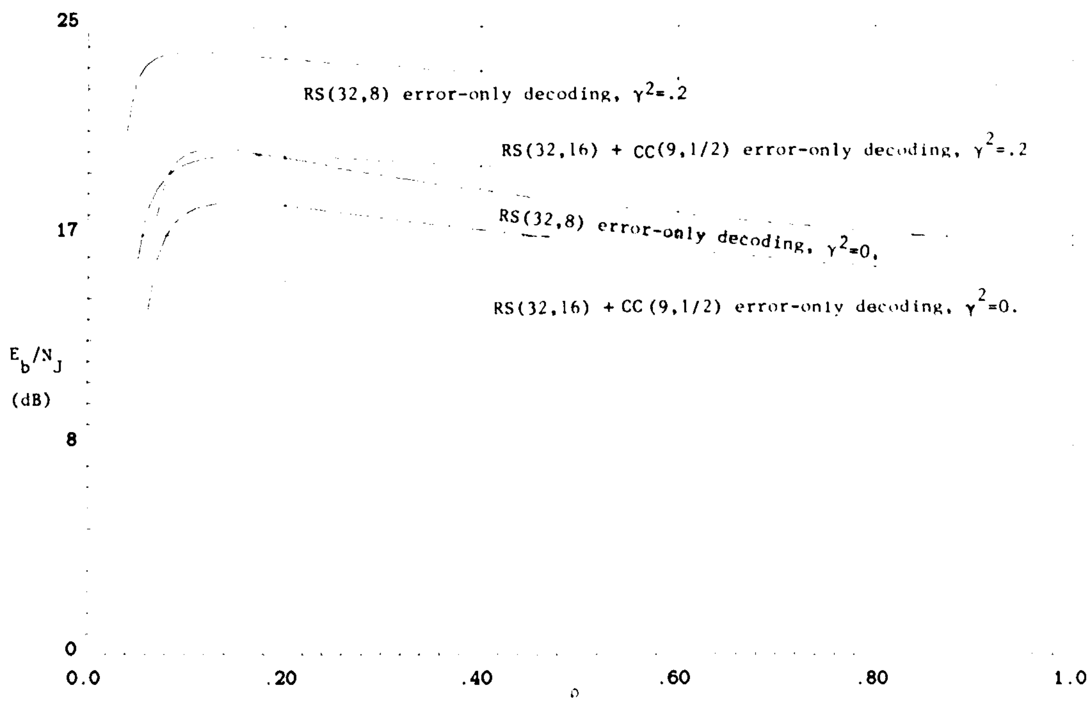


Fig. 15 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using binary FSK and various rate 1/4 coding schemes ($q = 100$, $N_b = 12$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel.

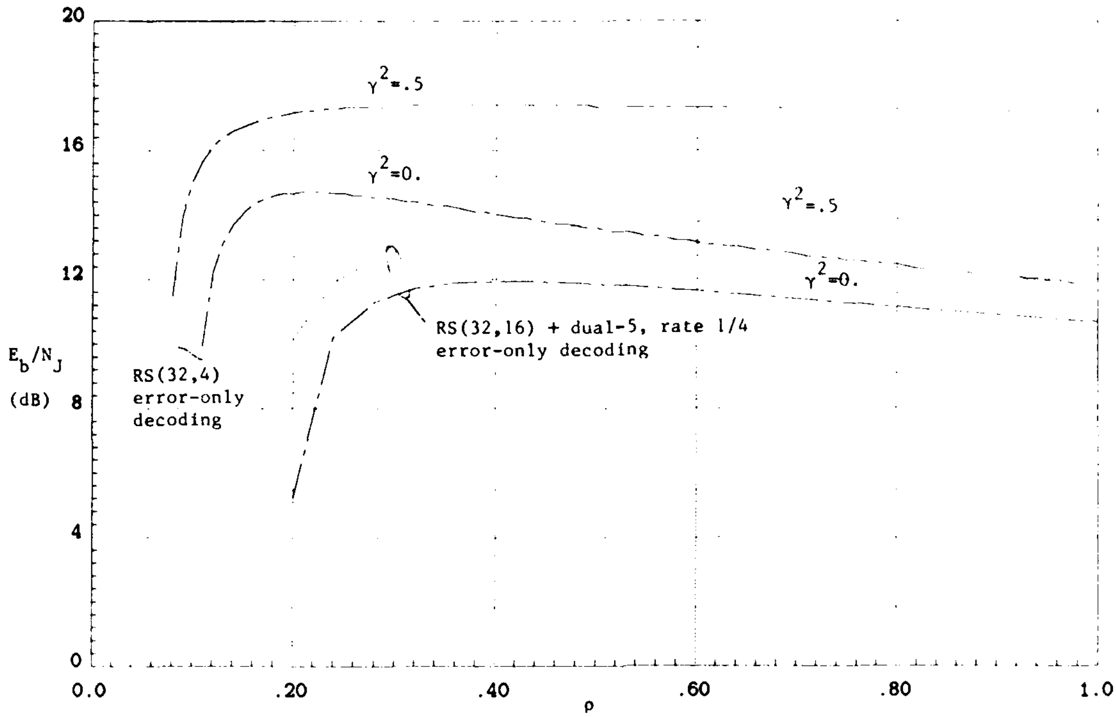


Fig. 16 — Minimum E_b/N_J required for $P_e = 10^{-5}$ vs ρ for asynchronous FH/SSMA communications using 32-ary FSK and various rate 1/8 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20$ dB, $K = 5$); AWGN channel, and Rician fading channel with $\gamma^2 = .5$.

Similarly, Fig. 16 compares the performance of the RS(32,4) code with that of the concatenated RS(32,16) + (dual-5,rate 1/4) code with overall rate 1/8 when error-only decoding and 32-ary FSK modulation are used. The figure shows that the concatenated scheme offers a substantial improvement in both the aforementioned performance measures; the improvement is actually more considerable than that of the concatenated scheme of Fig. 15.

The first group of results ends with Table 1. In this table, we cite the system performance indicators $(E_b/N_J)_{max}$ and ρ^* for a FH/SS system with 32-ary FSK modulation (unless indicated otherwise), noncoherent demodulation, and other system parameters as described in the table caption. The desirable values of the bit error probability are 10^{-3} and 10^{-5}. From the coding schemes presented, the RS codes with diversity show the best performance in particular, when we seek to maximize ρ^*. Recall that Figs. 15 and 16 show that concatenated codes provide very good performance for $(E_b/N_J)_{max}$ but not very good performance for ρ^*.

The second group of results starts with Tables 2 and 3. These tables show the multiple-access capability (K_{max}) of FH/SS systems that use 32-ary FSK modulation [except for the binary and nonbinary convolutional codes (CC) that use binary, 4-ary, and 8-ary FSK modulation] with noncoherent demodulation and a variety of error-control coding schemes. Table 2 assumes that $\rho = 0$ and $E_b/N_0 = \infty$, so multiple-access interference is the only source of interference. Therefore, K_{max} is the absolute maximum multiple-access capability of the FH/SS system. For RS codes and RS codes with diversity, we cite two numbers in each entry corresponding to erasure/error and parallel decoding, respectively. We see from the results of this table that the RS codes are superior for supporting a large number of users to most of the other codes of the same rate. Table 3 gives results similar to those of Table 2 for the case in which the fraction of the band jammed by the jammer is $\rho = 50\%$ and $E_b/N_J = 10$ dB. Observations similar to those of Table 2 are in order here.

Table 1 — Maximum value of signal-to-jammer power ratio $(E_b/N_J)_{max}$ (dB) required and ρ^* for an FH/SS system using 32-ary FSK with noncoherent demodulation (K = 5 asynchronous users, q = 100, N_b = 10, and AWGN with E_b/N_0 = 20 dB)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	--- (0.00)	--- (0.00)	--- (0.00)	--- (0.05)
Repetition code L=4	--- (0.00)	26.58 (0.01)	--- (0.00)	12.93 (0.05)
Repetition code L=5	--- (0.00)	15.28 (0.06)	--- (0.05)	11.56 (0.21)
Repetition code L=7	21.32 (0.02)	12.30 (0.16)	14.72 (0.15)	10.80 (0.34)
Repetition code L=9	16.44 (0.08)	11.65 (0.24)	13.65 (0.24)	10.86 (0.44)
RS(32,8)	13.71 (0.07)	10.34 (0.14)	10.47 (0.37)	8.91 (0.48)
RS(32,24)+diversity 3	17.36 (0.03)	12.09 (0.09)	10.87 (0.34)	9.17 (0.44)
RS(32,16)+diversity 3	12.55 (0.13)	10.52 (0.20)	10.33 (0.56)	9.30 (0.63)
RS(32,4)	14.72 (0.11)	12.15 (0.19)	12.22 (0.52)	10.97 (0.64)
RS(32,16)+diversity 4	11.09 (0.24)	9.82 (0.32)	10.27 (0.65)	9.47 (0.71)
CC(7,1/2) M=2	--- (0.00)	--- (0.00)	--- (0.20)	12.33 (0.32)
CC(7,1/3) M=2	20.51 (0.03)	13.80 (0.10)	12.89 (0.39)	11.20 (0.52)
CC(9,1/2) M=2	--- (0.00)	--- (0.00)	14.45 (0.26)	11.55 (0.36)
CC(9,1/3) M=2	17.44 (0.05)	13.07 (0.12)	12.33 (0.45)	10.90 (0.56)
CC(7,1/2) M=4	--- (0.00)	22.88 (0.01)	12.64 (0.11)	9.71 (0.21)
CC(7,1/3) M=8	21.49 (0.01)	11.67 (0.09)	11.15 (0.14)	8.57 (0.26)

Table 2 — Maximum number of asynchronous users that can be supported by an FH/SS system using 32-ary FSK with noncoherent demodulation (q = 100, N_b = 10, ρ = 0, and AWGN with $E_b/N_0 = \infty$)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	0	2	2	8
Repetition code L=4	0	5	4	14
Repetition code L=5	2	8	8	20
Repetition code L=7	6	15	15	32
Repetition code L=9	10	21	22	42
RS(32,8)	9	14	34 34	47 47
RS(32,24)+diversity 3	6	11	32 32	42 42
RS(32,16)+diversity 3	13	18	57 57	70 70
RS(32,4)	11	17	52 52	71 71
RS(32,16)+diversity 4	21	28	72 72	86 86
CC(7,1/2) M=2	0	0	25	38
CC(7,1/3) M=2	5	9	49	70
CC(9,1/2) M=2	0	0	31	44
CC(9,1/3) M=2	6	10	57	78
CC(7,1/2) M=4	2	5	17	29
CC(7,1/3) M=8	5	11	25	45
Dual-5 Rate=1/2	0	3	3	6
Dual-5 Rate=1/3	4	8	8	15
Dual-5 Rate=1/4	8	15	14	24
Dual-5 Rate=1/5	13	21	20	32
Dual-5 Rate=1/6	17	26	26	40
Dual-5 Rate=1/7	21	31	32	46
Dual-5 Rate=1/8	25	36	37	53
Dual-5 Rate=1/9	29	40	42	59
Dual-5 Rate=1/10	32	44	47	64

Table 3 — Maximum number of asynchronous users that can be supported by an FH/SS system using 32-ary FSK with noncoherent demodulation (q = 100, N_b = 10, ρ = .5, E_b/N_J = 10 dB, and AWGN with $E_b/N_0 = \infty$)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	0	0	0	0
Repetition code L=4	0	0	0	0
Repetition code L=5	0	0	0	0
Repetition code L=7	0	0	0	3
Repetition code L=9	0	0	0	8
RS(32,8)	1	6	0 4	1 11
RS(32,24)+diversity 3	0	3	0 3	0 9
RS(32,16)+diversity 3	0	4	12 14	24 26
RS(32,4)	0	0	6 8	25 26
RS(32,16)+diversity 4	1	8	26 28	40 41
CC(7,1/2) M=2	0	0	0	1
CC(7,1/3) M=2	0	0	4	21
CC(9,1/2) M=2	0	0	0	5
CC(9,1/3) M=2	0	0	9	27
CC(7,1/2) M=4	0	1	0	7
CC(7,1/3) M=8	0	4	0	18
Dual-5 Rate=1/2	0	2	0	3
Dual-5 Rate=1/3	1	5	2	7
Dual-5 Rate=1/4	1	7	3	11
Dual-5 Rate=1/5	1	9	4	14
Dual-5 Rate=1/6	2	11	6	18
Dual-5 Rate=1/7	3	13	8	21
Dual-5 Rate=1/8	4	15	11	25
Dual-5 Rate=1/9	6	17	13	28
Dual-5 Rate=1/10	7	19	16	32

In Figs. 17 to 21, we cite some values of K_{max} as functions of ρ for different error-control coding schemes and an AWGN channel. Figure 17 shows the increase in multiple-access capability (K_{max}) as the available signal-to-jammer power ratio (E_b/N_J) increases by 5 dB steps. The error-control code used is a binary convolutional code of constraint length 9 and code rate 1/3; side information is available.

Figure 18 shows the performance of binary convolutional codes with different constraint lengths and code rates. Codes with larger constraint lengths and lower code rates give uniformly (over all values of ρ in [0,1]) better performance than codes of smaller constraint lengths or higher code rates.

Figure 19 shows the performance of dual-5 convolutional codes of different code rates ($= 1/v$) using 32-ary FSK modulation. Both situations are considered when side information is available and when it is not available. The multiple-access capability increases as the code rate decreases from 1/2 to 1/3 and then to 1/4; it also increases when side information is available.

Figure 20 shows results similar to those of Fig. 17 for an RS(32,16) code using 32-ary FSK modulation and error-only decoding for varying E_b/N_J. As E_b/N_J increases, the multiple-access capability becomes insensitive to its actual value since the other-user interference dominates the jamming interference which then becomes negligible.

Finally, Fig. 21 shows the multiple-access capability vs ρ for several RS-coded FH/SS systems. For small values of ρ, RS(32,16) with diversity 4 outperforms all the other systems; in this range of ρ, the other-user interference is dominant. For large values of ρ, the RS(32,8) code outperforms the other schemes; in this range of ρ the jamming interference is dominant.

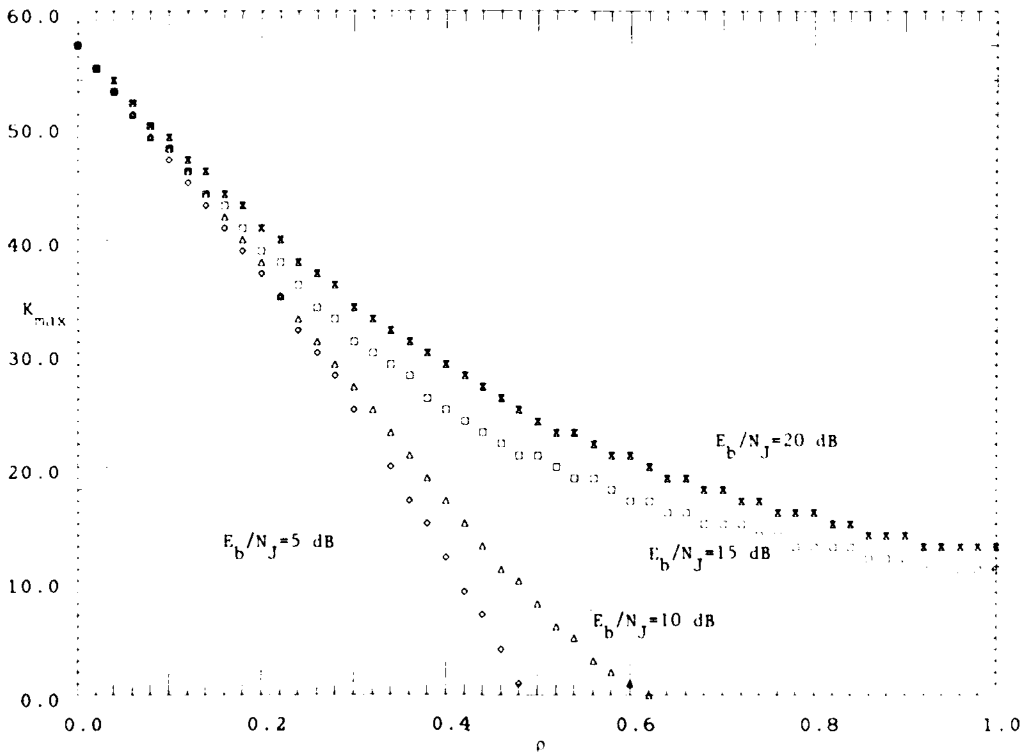


Fig. 17 — K_{max} vs ρ for asynchronous FH/SSMA communications using binary FSK and CC(9, 1/3) coding with side information for varying E_b/N_J (q = 100, N_b = 12, E_b/N_0 = 20 dB, AWGN, $P_e = 10^{-5}$)

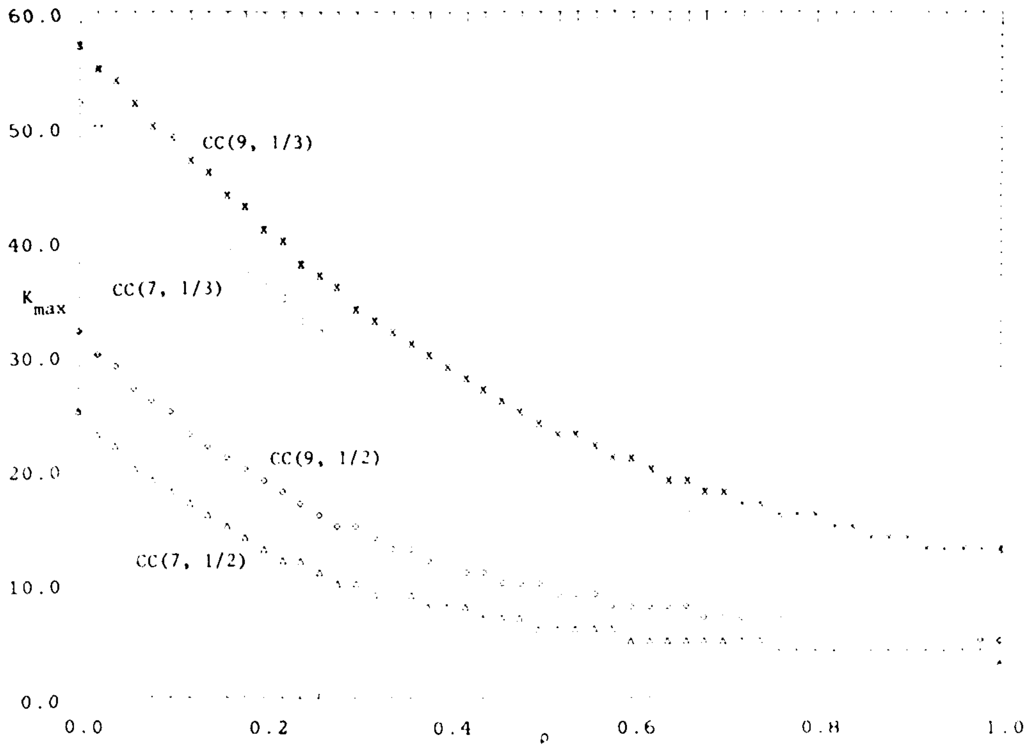


Fig. 18 — K_{max} vs ρ for asynchronous FH/SSMA communications using binary FSK with various convolutional coding schemes with side information (q = 100, N_b = 12, E_b/N_0 = 20 dB, E_b/N_J = 20 dB, P_e = 10^{-5}); AWGN channel.

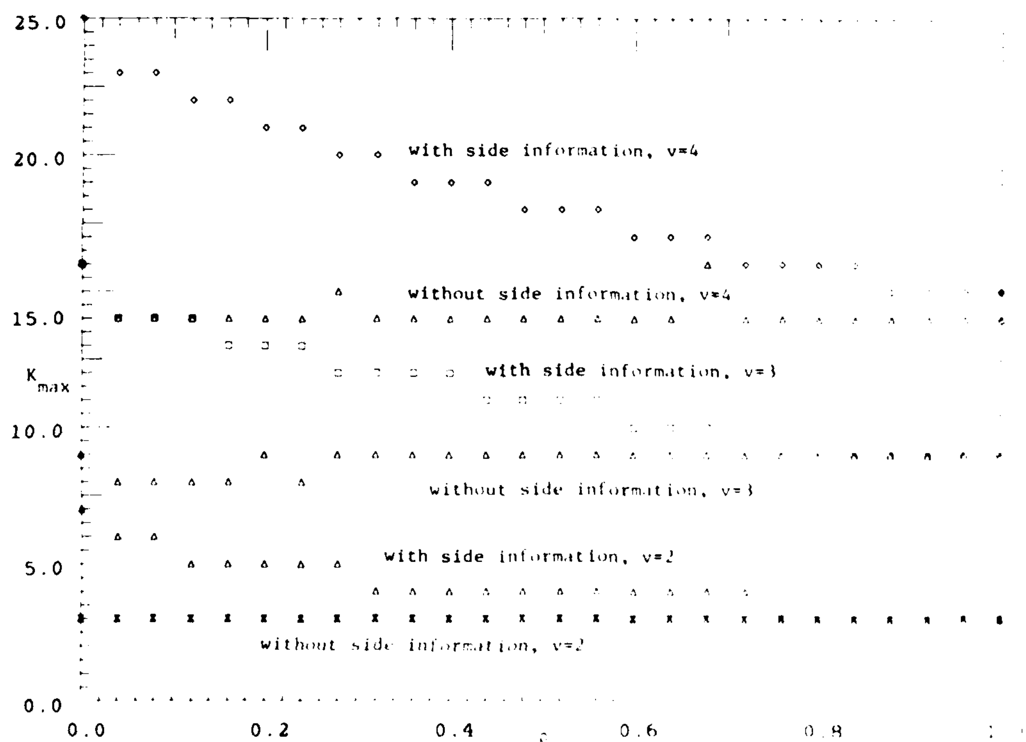


Fig. 19 — K_{max} vs ρ for asynchronous FH/SSMA communications using 32-ary FSK with various dual-5 coding schemes (q = 100, N_b = 10, E_b/N_0 = 20 dB, P_e = 10^{-5}), AWGN channel

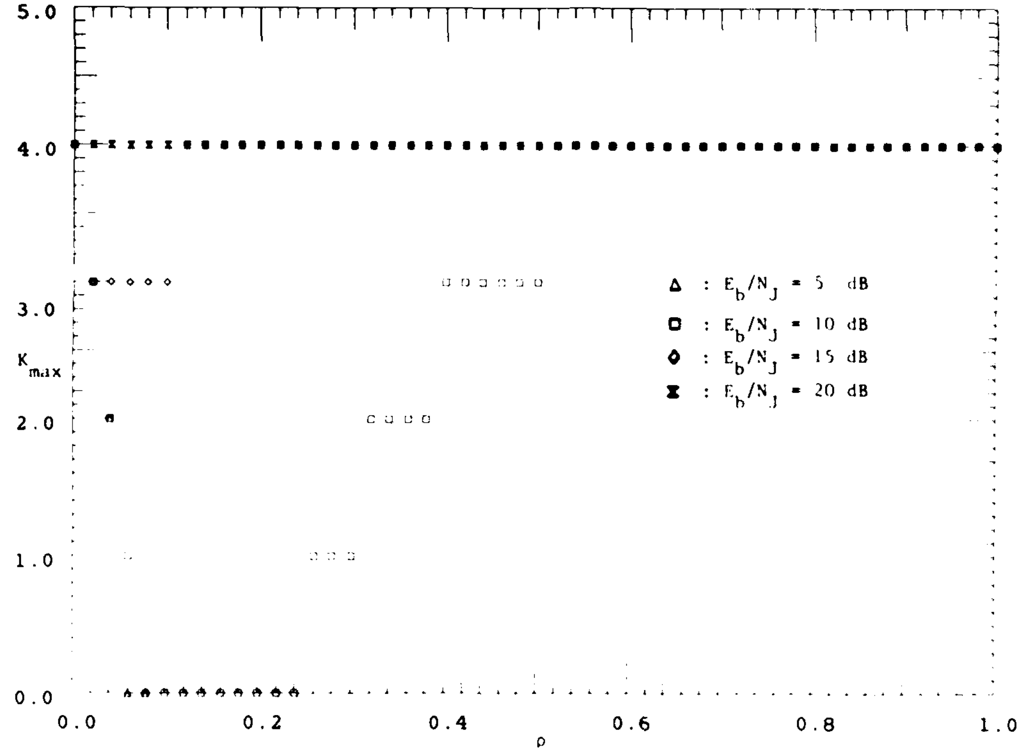


Fig. 20 — K_{max} vs ρ for asynchronous FH/SSMA communications using 32-ary FSK and *RS* (32,16) coding with error-only decoding and varying E_b/N_J (q = 100, N_b = 10, E_b/N_0 = 20 dB, AWGN, $P_e = 10^{-5}$).

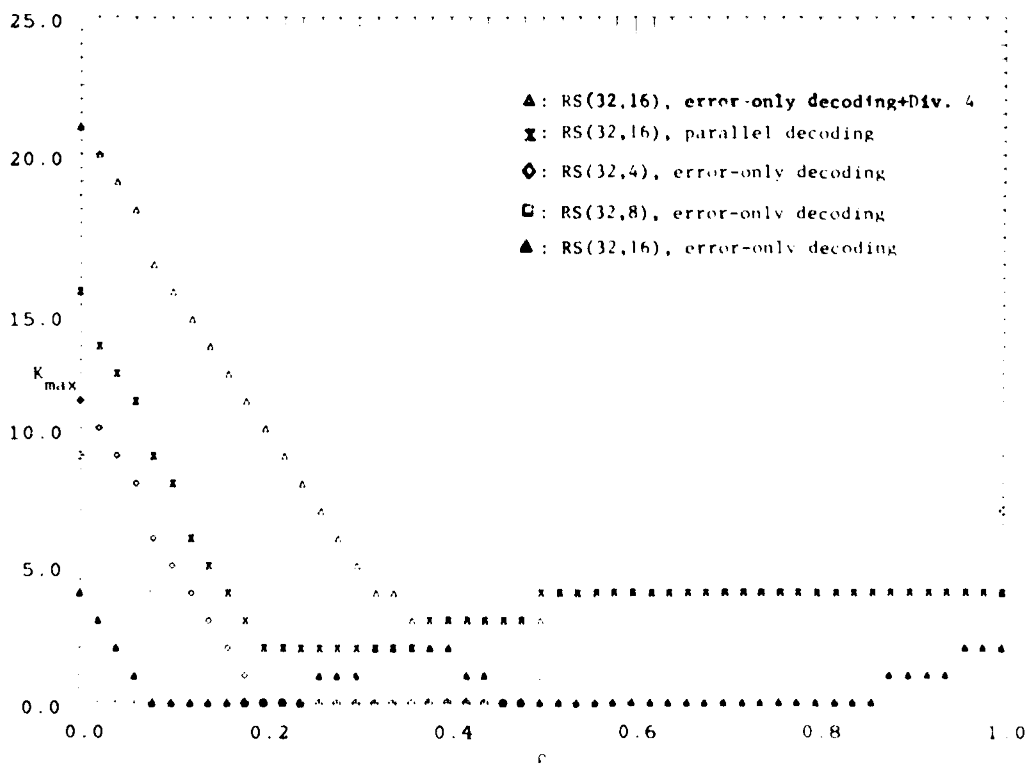


Fig. 21 — K_{max} vs ρ for asynchronous FH/SSMA communications using 32-ary FSK and Reed-Solomon coding with various rates and decoding methods, with and without diversity (q = 100, N_b = 10, E_b/N_0 = 20 dB, AWGN, E_b/N_J = 10 dB, $P_e = 10^{-5}$).

5. CONCLUSIONS

This report provides an analytical framework for evaluating the performance of coded FH/SS systems operating in the presence of combined partial-band noise jamming, Rician nonselective fading, other-user interference, and AWGN. Several forward error-control coding schemes with or without side information at the decoder, with binary or nonbinary data modulation schemes, as well as combinations of coding schemes, have been analyzed. Numerical results have been presented for several cases and can be easily obtained from the available formulas for all cases for which they were not presented.

Our conclusions are the following. For the combined interference considered in this paper, Reed-Solomon codes perform better than convolutional codes of the same rate when hard decisions are used; no soft decision decoding was considered in this report. Similarly, RS codes outperform the repetition codes for code rates not lower than a critical rate (usually 1/4), a situation that is reversed for code rates lower than the critical rate. Using RS codes with diversity and parallel decoding (when side information is available) provides excellent performance in terms of both performance measures, $(E_b/N_J)_{\max}$ and ρ^*

The availability and use of side information improves the system performance in all cases; in particular, it increases the value of ρ^* more drastically than it decreases the value of $(E_b/N_J)_{\max}$. In contrast, increasing the value of M (i.e., using codes with nonbinary alphabets) decreases the value of $(E_b/N_J)_{\max}$ more drastically than it increases the value of ρ^*. Similarly, lowering the code rate improves ρ^* more drastically than it improves $(E_b/N_J)_{\max}$.

In the absence of side information, the use of concatenated codes, especially of RS outer codes (using parallel decoding) with inner parity-check codes (to detect the presence of interference), retrieves most of the advantage that perfect knowledge of side information offers with respect to $(E_b/N_J)_{\max}$; to increase ρ^* we also need to lower the rate of the outer code. When RS outer codes (using error-only decoding) with nonbinary (preferably dual-k) convolutional inner codes are used, lowering the rate of the inner code retrieves most of the advantage that perfect side information offers with respect to both ρ^* and $(E_b/N_J)_{\max}$.

6. REFERENCES

1. E. A. Geraniotis and M. B. Pursley, "Error probabilities for slow frequency-hopped spread-spectrum multiple access communications over fading channels," *IEEE Trans. Comm.* **COM-30**, 996-1009 (1982).

2. M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt, *Spread-Spectrum Communications* (Computer Science Press, 1985).

3. M. B. Pursley, "Coding and diversity for channels with fading and pulsed interference," in *Proc. 1982 Conf. Inform. Sci. Syst.*, March 1982, pp. 413-418.

4. W. E. Stark, "Coding for frequency-hopped spread-spectrum channels with partial-band interference," Ph.D dissertation, Elect. Eng. Dept., Univ. of Illinois and Coordinated Science Laboratory Technical Report R-945, July 1982.

5. M. B. Pursley and W. E. Stark, "Performance of Reed-Solomon coded frequency-hop spread-spectrum communications in partial-band interference," *IEEE Trans. Comm.* **COM-33**, 767-774 (1985).

6. W. E. Stark, "Coding for frequency-hopped spread-spectrum communication with partial-band interference - Part II," *IEEE Trans. Comm.* **COM-33**, 1045-1057 (1985).

7. R.-H. Dou and L. B. Milstein, "Erasure and error correction decoding algorithm for spread-spectrum systems with partial-time interference," *IEEE Trans. Comm.* **COM-33**, 858-862 (1985).

8. A. J. Viterbi and I. M. Jacobs, "Advances in coding and modulation for noncoherent channels affected by fading, partial band and multiple-access interference," in *Advances in Communication Systems*, Vol. 4. (Academic Press, New York, 1975), pp. 279-308.

9. A. M. Michelson and A. H. Levesque, *Error-Control Techniques for Digital Communication* (John Wiley and Sons, New York, 1985).

10. M. B. Pursley, unpublished note, 1981

11. E. R. Berlekamp, "The technology of error-correcting codes," *Proc. IEEE* **68**, 564-593 (1980).

12. G. C. Clark, Jr. and J. B. Cain, *Error-Correction for Digital Communication* (Plenum, New York, 1981).

13. J. Conan, "The weight spectra of some short low-rate convolutional codes," *IEEE Trans. Comm.* **COM-32**, 1050-1053 (1984).

14. J. P. Odenwalder, "Dual-k convolutional codes for non-coherent demodulated channels," *Proc. IEEE Int. Telem. Conf.*, 1976, pp. 165-176.

15. R. J. McEliece, *The Theory of Information and Coding* (Addison-Wesley, Reading, MA, 1977) (Problem 7.10).

16. W. E. Stark, "Performance of concatenated codes on channels with jamming," *1982 International Conf. on Comm.*, Conf. Record, pp. 7E.4.1-5, June 1982.

17. F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes* (North-Holland, Amsterdam, The Netherlands: 1977).

END
9-87
DTIC